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Routes to chaos of natural convection flows in vertical channels

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ABSTRACT

The aim of the present study is the analysis of the transition to turbulence of natural convection flows between two infinite vertical plates. For the study of the problem, a number of Direct Numerical Simulations (DNSs) have been performed. The continuity, momentum and energy equations, cast under the Boussinesq assumption, are tackled numerically by means of a pseudospectral method, through which the three-dimensional domain is decomposed with Chebychev polynomials in the wall-normal direction and with Fourier modes in the wall-parallel directions. For low Rayleigh number values, the predictions of the flow regimes are consistent with the classical analytical results and linear stability analyses. In particular, the first bifurcation ($Ra \approx 5800$) from the so-called laminar conduction regime to steady convection is correctly captured. By increasing the Rayleigh number beyond a second critical value ($Ra \approx 10200$), the flow regime becomes chaotic. This transition to chaos is found to be related with the amplification of spanwise instabilities occurring at scales larger than the channel gap, H. The study of the return of the system from the chaotic regime to the laminar base flow reveals a phenomenon of hysteresis, i.e. the chaotic regime persists even at Ra-values lower than the second critical value. From a numerical point of view, the predicted flow regimes appear to be extremely sensitive to the domain size, grid resolution and perturbation amplitude. These aspects are shown to be of crucial importance for the prediction of the heat transfer performance, and, hence, should be taken into consideration when numerical methods are used for the simulation of real-world problems.

1. Introduction

Transition to chaos of natural convection in a fluid layer between two differentially heated vertical plates is a topic of substantial interest for many civil applications involving energy management. Among these, notable examples are the air gaps in double-glazing panes or in naturally ventilated double-skin façades. The correct prediction and control of flow regimes, air flow rates and heat transfer coefficients have a significant impact in the correct design of such elements and, in turn, on the energy balance of buildings.

Contrarily to the case of a horizontal fluid layer heated from below (the well-known Rayleigh-Bénard convection problem), where the fluid remains stagnant until a certain temperature gradient is established, in a differentially heated vertical layer, fluid motion ensues as soon as a temperature difference is applied across the two plates. However, for low values of the leading parameter (the Grashof number, or the Rayleigh number) the intensity of the flow remains feeble, and heat transfer between the two walls occurs only by thermal diffusion. This particular situation can be described analytically, and a steady-state solution of the governing equations can be found [1], corresponding to the commonly named *conduction regime*. The conduction regime represents the desired condition for the correct operation of air-filled insulating gaps.

If the thermal input is increased beyond a certain threshold, the conduction regime loses its stability and convective transport sets on [2]. Vest and Arpaci [3] were the first to tackle the problem of the stability of the conduction regime, finding that the onset of convection occurs for a critical value of the Grashof number $Gr_c \approx 7880$, almost independently of the Prandtl number, for a wide range of Pr-values. The instability determines the appearance of a new stable flow solution, in the form of steady-state co-rotating convective rolls, arranged in a periodic pattern and lined up along the vertical direction. A similar study was conducted in the very same period by Gill and Davey [4], leading to an estimate of the critical Gr-value very close to the one of Vest and Arpaci. Later, Gill and Kirkham [5] explored the stability of the system for large values of the Prandtl number, and discovered that, for $Pr > 10^2$, the instability occurs for lower values of Gr, and manifests itself as a traveling wave, giving rise to an unsteady, periodic flow. These findings were later refined by the numerical work of Korpela et al. [6]. More recently,

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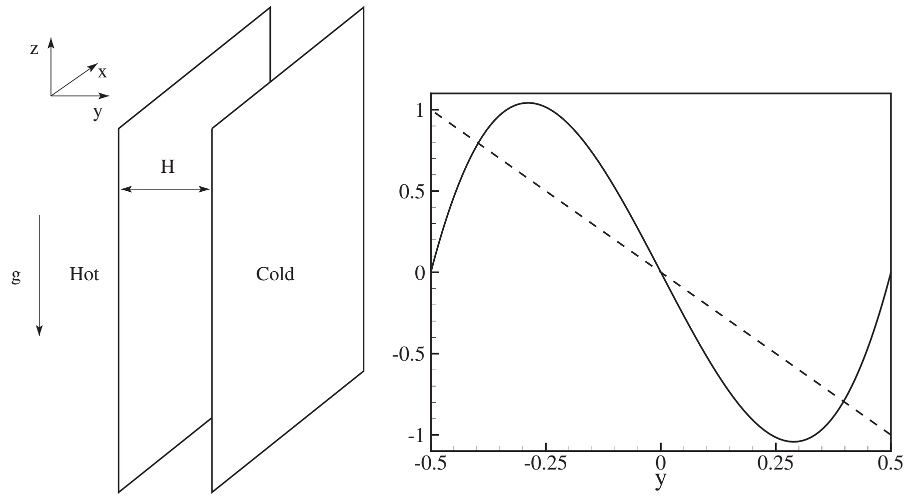

Fig. 1. Left: setup of vertical natural convection. Right: base flow profiles at $Ra = 3000$. The temperature profile is shown with dashed line and velocity profile with solid line.

McBain and Armfield [7] reconsidered the problem and obtained a large set of accurate numerical solutions for the linear stability problem, thus determining marginal stability curves covering the entire range of Pr-values, including the asymptotic limits $Pr \to 0$ and $Pr \to \infty$. A thorough review of linear and nonlinear stability studies on the case can be found in a recent paper by Barletta [8].

Turbulent convection regimes were also the subject of many studies. Versteegh and Nieuwstadt reported direct numerical simulation data aimed at determining scaling laws and wall functions for turbulence modelling [9] and at understanding the main physical processes of the flow in terms of turbulence budgets [10]. Further developments of the inner (near-wall) and outer (bulk) scaling can be found in Hölling and Herwig [11] and Ng et al. [12]. Regarding turbulence modelling, various proposals for the Reynolds-averaged modelling are analysed and new models are proposed in Dol et al. [13]. Finally, several works have been also devoted to the study of possible scaling laws correlating the Nusselt number with the Rayleigh and Prandtl numbers. As an example, in Ng et al. [14] it is argued that the underlying physics of vertical natural convection leads to non-pure power laws for the Rayleigh, Prandtl and Nusselt numbers.

In this context, much less work has been carried out so far to investigate the evolution of the system in between the above-described first transition [15,16] and the onset of turbulence. To date, the first and only real attempt at a systematic analysis of the transition from steady-state to unsteady, chaotic flow is contained in the numerical study of Gao et al. [17], carried out for $Pr = 0.71$, in which the occurrence of successive bifurcations is discussed, leading the system to chaos through 3D steady-state and unsteady, periodic flow. Traces of a period-doubling cascade were also reported, although the subharmonic patterns were observed only in transient windows. As acknowledged by the authors, though, the main limitation of their analysis consisted in the adoption of a relatively small domain in the transversal direction, which could have prevented the onset of spanwise instabilities.

Table 1
Parameters of the simulations.

Case	$L_x \times L_y \times L_z$	$N_x \times N_y \times N_z$	Δ_x	Δ_y^{max}	Δ_z
LDS	$8H \times H \times 16H$	$256 \times 129 \times 256$	0.031H	0.012H	0.062H
SDS	$H \times H \times 16H$	$32 \times 129 \times 256$	0.031H	0.012H	0.062H
SDSlr	$H \times H \times 16H$	$32 \times 49 \times 256$	0.031H	0.032H	0.062H

The aim of this work is to study the problem of transition to chaos in an air-filled, differentially-heated vertical layer, by means of direct numerical simulations. The produced data set allows for a detailed description of the flow patterns taking place during the sequence of instabilities leading to chaos. The physical understanding of these coherent motions is very important from an energetic point of view since these flow structures are responsible for driving the heat exchange in such a system and, hence, the energy efficiency of similar applications.

2. Direct Numerical Simulations

Different Direct Numerical Simulations (DNSs) have been performed varying the Rayleigh number, the domain size and the numerical resolution. The governing equations of the problem are

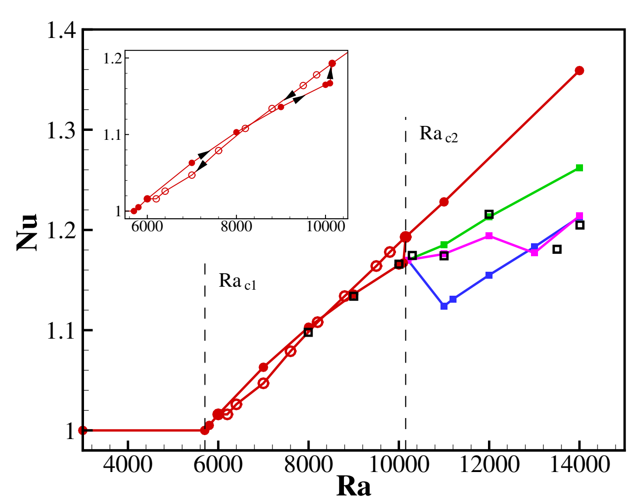

Fig. 2. Behavior of the Nusselt number as a function of the Rayleigh number. LDS forward branch (filled red circles), LDS backward branch (empty red circles), SDS (blue squares), SDSlr (green squares) and SDS with finite amplitude disturbance (magenta squares). The empty black squares show the behavior of Nu reported in Gao et al. [17]. The inset plot is a zoom of the forward and backward branch of the LDS case where arrows indicate the path's direction.