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Flow and heat transfer of power-law fluid over a rotating disk with generalized diffusion☆

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ABSTRACT

The study of swirling flows and heat transfer near various rotating machines, such as fans, turbines and centrifugal pumps, is necessary and important for many manufacturing processes in industry, especially the cooling of turbojet engines. The flow and heat transfer of power-law fluids over an infinite rotating disk is investigated in this paper. A generalized Fourier heat transfer model is introduced in which the thermal conductivity is assumed to depend on temperature gradient. New similarity variables are defined and the governing equations in the boundary layer are reduced to a set of coupling ordinary differential equations. An improved multi-shooting method is proposed to solve the resulting singular boundary value problems. The effects of the power-law index and local Prandtl number on velocity, pressure and temperature fields are analyzed. Especially, the viscosity coefficient and heat conductivity are discussed.

1. Introduction

Flows and heat transfer due to rotating disks are important in theory and in many practical engineering applications, such as the design of the gas turbine rotors and electronic devices with rotary parts. Steady laminar flows of viscous Newtonian fluid over an infinite rotating disk were studied originally by von Karman [1], who proposed an elegant similarity transformation which reduces the Navier-Stokes equations to ordinary differential equations, which were then solved by the momentum integral method. Cochran [2] gave a more accurate asymptotic series solution to Karman's viscous pumping flow. Karman swirling flow has received considerable attention over the years [3]. The heat transfer of Karman swirling flow was considered by Millsaps and Pohlhausen [4] and Sparrow and Gregg [5]. Dorfman [6] considered a thermal boundary condition of power-law distribution for the wall temperature for a free rotating disk. Shevchuk [7] gave a new analytical solution with Nusselt number being specified as a boundary condition in the form of an arbitrary power-law function, and the results were compared with experimental data. Hayat et al. [8] and Frusteri and Osalusi [9] investigated three-dimensional flows in which thermal conductivity depends on temperature. More investigations on the swirling flow and heat transfer with the aid of Karman's similarity transformation have been reported [10–15]. Another extension of Karman swirling flow is to a case in which the spinning disk is rotating in a non-Newtonian rather than Newtonian fluid. Many fluids are non-Newtonian, for which a linear relationship between the stress tensor and the deformation tensor is not satisfied, such as blood, petroleum, slurry, emulsion, pulp, etc. A detailed discussion up to 1991 on flows of non-Newtonian fluids due to rotating disks can be found in the review paper by Rajagopal [16]. Ariel presented a research on rotating flows of elastic-viscous Oldroyd B fluid [17] and second-grade fluid [18]; perturbation solutions for small non-Newtonian fluid parameter and asymptotic analytical solutions for large parameter were obtained. Siddiqui et al. [19] studied the heat transfer on the magnetohydrodynamic (MHD) flow of Burgers' fluid between two disks, and the effects of Hartmann number, Prandtl number, Eckert number and Hall parameter were analyzed. Sahoo et al. researched the flow of Reiner-Rivlin fluid over a rotating disk with heat transfer [20] and the effects of the non-Newtonian fluid characteristics on the velocity and temperature distributions as well as the heat transfer were considered [21]. The swirling flow and heat transfer of Bingham fluid were investigated by Rashaida et al. [22]. Osalusi et al. [23] examined the effects of viscous dissipation and Joule heating on steady MHD flow of a Bingham fluid over a porous rotating disk in the presence of Hall and ion-slip currents. Attia studied the unsteady flow and heat transfer of Reiner-Rivlin fluid over a rotating disk by finite difference method [24] and investigated the effect of suction on the flow and heat transfer [25]. Sheikholeslami et al. [26–31] researched the flow and heat transfer of magnetic nanofluid in various situations. In order to deal with the three-dimensional swirling flow over a rotating disk for power-law fluid, Mitschka [32] proposed generalized Karman similarity transformations. Andersson and Korte [33] reviewed Mitschka's work and obtained velocity distributions for

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Nomenclature

Roman symbols

c_p	specific heat at constant pressure (J/(kg·K)
F	self-similar radial velocity
G	self-similar tangential velocity
H	self-similar axial velocity
K	consistency coefficient of the fluid
n	power law index
p	pressure (N/m^2)
Pr	Prandtl number
Q	self-similar pressure function
r	radial coordinate (m)
R	characteristic length (m)
Re	Reynolds number
T	temperature in the flow region (K)
U	reference velocity (m/s)
u	radial velocity component (m/s)
v	tangential velocity component (m/s)
w	axial velocity component (m/s)
z	normal coordinate (m)
$\bar{r}, \bar{z}, \bar{u}, \bar{v}, \bar{w}, \bar{p}, \bar{T}$	dimensionless function

Greek symbols

α	radial velocity gradient on the disk
β	tangential velocity gradient on the disk
φ	angular coordinate (Rad)
γ	axial velocity gradient on the disk
κ	thermal conductivity (W/(m·K))
λ	heat coefficient
μ	coefficient of viscosity (Pa·s)
ρ	density (kg/m^3)
σ	thermal gradient on the disk
τ	wall shear stress (N/m^2)
ξ	dimensionless normal distance
Θ	dimensionless temperature function
Ω	angular speed of the disk (Rad/s)

Subscripts

w	wall ($z = 0$)
∞	infinity

different power-law indices in the range $0.5 \le n \le 2$ by a finite difference method, and extended the work to MHD flow [34]. Denier and Hewitt [35] considered the pressure correction and pointed out that there is a critical singular position for shear-thickening fluid. Griffiths [36] analyzed the stability of the rotating flow for shear-thinning fluid by a linear stability analysis.

As far as the heat transfer of power-law fluid over a rotating disk is concerned, little work has been done because no suitable similarity solution has been found for the energy equation. In a previous work [37], we obtained the similarity solutions for the flow and thermal boundary layer equations of power-law fluids over a rotating disk under the assumption that the thermal conductivity of non-Newtonian fluids depends on the velocity gradient. This assumption was proposed in Pop et al. [38,39]. In the present work, the pressure and heat transfer of power-law fluids over a rotating disk are investigated under the assumption that the thermal conductivity depends on the temperature gradient. This assumption is motivated by the assertion that surface tension is a function of temperature, and the effects of power-law fluid viscosity on the temperature field can be taken into account by assuming that the temperature field behaves in a similar way to the velocity field. In this setting, Zheng et al. [45] studied flow and heat transfer of Marangoni convection of a power-law non-Newtonian fluid due to power-law temperature gradients. A significant outcome of the work in [45] was that the temperature and the thermal boundary layer decrease as the power-law number and the Marangoni number increase for non-Newtonian fluids. We will explore Zheng's model [45] further in this paper to study swirling flow and heat transfer of power-law fluids over an infinite rotating disk.

This paper is organized as follows. First the governing equations are formulated in Section 2. The similarity transformation is given, and the governing equations including the energy equation in the boundary layer are recast as a set of ordinary differential equations in Section 3. The resulting system of highly nonlinear differential equations for the velocity, pressure and temperature field is solved by an improved multi-shooting method, and numerical results are shown in Section 4. Finally some conclusions are drawn in Section 5.

2. Governing equations

We consider a steady laminar flow driven by an infinite disk rotating with constant angular velocity Ω about the z-axis. The cylindrical coordinate is (r,φ,z), and (u,v,w) are the velocity components along the (r,φ,z) directions, respectively. The fluid is thrown out and the upper fluid falls down when the disk is rotating. Heat transfer is considered. Let T be the temperature of the fluid. The motion and heat transfer of the incompressible fluid above the disk is governed by the conservation equations for mass, momentum and energy:

$$\frac{1}{r}\frac{\partial(ru)}{\partial r}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial r}-\frac{v^2}{r}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial r}+\frac{2}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right)+\frac{\partial}{\partial z}\left(\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right)\right)-2\mu\frac{u}{r^2} \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial r}+\frac{uv}{r}+w\frac{\partial v}{\partial z}\right)=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^3\mu\frac{\partial}{\partial r}\left(\frac{v}{r}\right)\right)+\frac{\partial}{\partial z}\left(\mu\frac{\partial v}{\partial z}\right) \tag{3}$$

$$\rho\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right)\right)+2\frac{\partial}{\partial z}\left(\mu\frac{\partial w}{\partial z}\right) \tag{4}$$

$$\rho c_p\left(u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\right)=\frac{\partial}{\partial r}\left(k\frac{\partial T}{\partial r}\right)+\frac{k}{r}\frac{\partial T}{\partial r}+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) \tag{5}$$

where μ is the dynamic viscosity, p is the fluid pressure, ρ is the fluid density, c_p is the specific heat at constant pressure of the fluid, k is the thermal conductivity.

The ambient fluid far away from the disk is assumed to be at rest. The temperature maintains at constant T_w on the disk surface and keeps a uniform temperature T_∞ out of the boundary layer. p_w is the pressure on the disk surface. The associated boundary conditions are

$$u=0, v=\Omega r, w=0, p=p_w, T=T_w \quad \text{at } z=0 \tag{6}$$

$$u=0, v=0, T=T_\infty \quad \text{as } z\to\infty \tag{7}$$

The fluid obeys the Ostwald-de Waele power-law model in which the viscosity can be expressed as

$$\mu=K\left[2\left(\frac{\partial u}{\partial r}\right)^2+2\left(\frac{u}{r}\right)^2+2\left(\frac{\partial w}{\partial z}\right)^2+\left(\frac{\partial w}{\partial r}+\frac{\partial u}{\partial z}\right)^2+\left(r\frac{\partial}{\partial r}\left(\frac{v}{r}\right)\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right]^{\frac{n-1}{2}} \tag{8}$$

where K is the consistency coefficient of the fluid and n is the power-law index with $0<n<1$, $n=1$ and $n>1$ representing shear-thinning (pseudoplastic) fluid, Newtonian fluid and shear-thickening (dilatant) fluid, respectively.

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