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Numerical analysis of developed tip leakage cavitating flows using a new transport-based model☆

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ABSTRACT

A new cavitation model, which takes into account the effects of vortex on mass transfer process in cavitation, is utilized for the computations of the developed tip leakage vortex cavitating flows. Compared with the conventional Zwart's model, better agreement is observed between the present model predicting cavity shape and the results of the experiments. Based on the computations, it is indicated that the rotating function in the cavitation source terms contributes to the improved modelling process of the liquid–vapor interface mass transfer. This is the main reason for the present model's better capability of predicting the cavity shape compared with the conventional Zwart's model. Furthermore, based on the predictions of the present model, it is found that as the decrease of the cavitation number, the leading edge attached cavity develops gradually and covers the suction side. This will result in the decrease of the hydrofoil lift and hence the TLV circulation.

1. Introduction

The tip vortex is a phenomenon common to all 3D lifting surfaces, including open and ducted rotors, control surfaces, and hydrofoils. High velocities found in the cores of tip vortex can lead to cavitation in the wake and cause noise, erosion, vibration, and performance decay [1–6]. When a tip is located at a finitely-small distance from an endwall, the vortex is called a tip leakage vortex (TLV). Extremely complex vortex flows, consisting of a primary TLV and numerous secondary co-rotating and counter-rotating vortices, have been observed experimentally in the gap and the wake [7–10]. Moreover, in the multiphase cavitating flows, vortex–cavitation interactions would greatly enhance the complexity of these vortices [11–15]. The need to understand and control the dynamics of developed TLV cavitating flows has driven numerous researchers [16–24].

Significant challenges still exist with respect to the accuracy, stability, efficiency and robustness of the modelling strategies because of the complex interactions associated with the TLV cavitating flows. Rains [25] first proposed a “jet model” for single phase tip leakage flows based on the slender body approximation. Higashi et al. [26], Watanabe et al. [27] and Murayama et al. [28] introduced the thickness of the hydrofoil and the bubble dynamics to the “jet model” to numerically predict the behaviour of the TLV cavitation of a rectangular hydrofoil. This series of modelling methods originates from the single phase flows, and hence the interactions between vortex structures and interface mass transfer are not included. Kinnas et al. [29,30] and Hanseong Lee [31] applied a low order potential based boundary element method for the numerical modelling of the developed tip vortex cavitation. It should be pointed out that a termination model must be applied at the end of the cavity, which is difficult to be physically defined in the developed tip vortex cavitating flows. The Navier–Stokes-based computations of turbulent cavitating flows have become quite popular due to advances in computational capabilities and in the physical modelling of cavitating problems. Among these approaches, the transport-based models are becoming increasingly popular because they include the physics of cavitating flows [32–38]. For example, the Zwart's model, which is proposed by Kubota et al. [32] and supplemented by Zwart et al. [38], as one of these typical transport-based models, has been well used to calculate the TLV cavitation [17,39,40]. However, its predictive capability has not been assessed.

Cavitation–vortex interactions have received increased attention as they play an important role in cavitating flows. Gopalan et al. [14] observed that the collapse of the vapor structure is a primary mechanism of vorticity production. Iyer et al. [12] and Laberteaux et al. [13] further pointed out that the baroclinic torques were responsible for the production of vorticity during vapor cloud collapse since density gradients within the cloud cavitation are not necessarily aligned with the pressure gradients around the cloud during collapse. Additionally,

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Nomenclature

p	Pressure
p_v	Saturation vapor pressure
p_∞	Reference pressure
ρ	Density
r_0	Radius of the central core
α	Volume fraction
t	Time
U	Velocity
U_∞	Reference velocity
L_0	Characteristic length scale
ϕ	Mixture property of fluids
μ	Dynamic viscosity
υ	Kinematic viscosity
S	Shear strain rate
Ω	Vorticity
k	Turbulence kinetic energy
ω	Specific dissipation rate
a_1, a_2, β^*, β_ω, F_1, F_2,σ_k, σ_ω, $\sigma_{\omega 2}$	Coefficients of k–ω turbulence model
$\Gamma(\Gamma^*)$	Circulation(dimensionless circulation)
m_B	Mass of a single bubble
N_B	Number of bubbles per unit volume
$\dot{m}^+, \dot{m}^-$	Source and sink terms in the cavitation model
C_{Cond}, C_{Vap}	Coefficients of the present cavitation model
$C_{Cond}^{\mathrm{Zwart's}}$,$r_0^{Const}$	Coefficients of Zwart's cavitation model
F_r	Function in the present cavitation model
f_r	Rotating function
C_{r1}	Coefficient of rotating function
r^*	Function in the present cavitation model
Re	Reynolds number
σ	Cavitation number
C	Chord length of hydrofoil
s	Span of hydrofoil
AOA	Angle of attack
$G(\tau)$	Gap size(dimensionless gap size)
L	Tunnel section length
H	Tunnel section height
B	Tunnel section breadth
X, Y, Z	Space variables
$F(F^*)$	Force(dimensionless force)
δ	Kronecker Delta function
y^+	Dimensionless wall distance

Subscript

i, j, k	Component
l	Liquid
v	Vapor
m	Mixture
L	Laminar
T	Turbulent

Dittakavi et al. [41] and Huang et al. [11,42] discussed the influence of cavitation on different terms of vorticity transport equation, and found that the baroclinic and the viscoclinic torques are important mechanisms for vorticity production and modification. Ji et al. [15,43] also found that even though the magnitude of the baroclinic torque term is smaller if compared with the vortex stretching term and dilatation term, the baroclinic torque is important for the production of vorticity and modifies the vorticity field along the liquid–vapor interface. It should be noted that these researches focused on the attached sheet/cloud cavitating flows, with limited attention paid to the developed TLV cavitation.

Hence, the goals of the present study are to

(1) assess the predictive capability of the present model and conventional model for the developed TLV cavitation, (2) study the modelling of liquid–vapor interface mass transfer in the TLV cavitation, (3) and discuss the influence of cavitation on vortex in the developed TLV cavitating flows.

2. Mathematical model

2.1. Vortex cavitation model

The authors proposed a new “vortex cavitation” model [44], which is utilized to simulate the developed TLV cavitating flows in the present study.

(a) Firstly, a cavitation vortex is defined to help build the relations between the cavitation bubble radius and vortex effects. Fig. 1 shows the comparisons of the pressure distributions of free and cavitation vortices. In the figure, p_v is the vapor pressure at the liquid temperature, p_∞ is the pressure of the surroundings, ρ is the density, Γ is the circulation of vortex, and r_0 is the radius of the bubble. For the free vortex, the pressure is continuous and decreases as the circulation (Γ) increases. For the cavitation vortex, however, the pressure in the central region ($r \le r_0$) remains constant. Moreover, the pressure at the bubble boundary ($r = r_0$) can be calculated as:

$$p_{bubble\_boundary} = p_v = p_\infty - \frac{\rho\Gamma^2}{8\pi^2}\frac{1}{r_0^2}. \tag{1}$$

Then, the radius of the cavitation bubble can be determined by:

$$r_0 = \frac{1}{2\pi}\sqrt{\frac{\rho}{2(p_\infty - p_v)}} \cdot |\Gamma|. \tag{2}$$

This equation shows that the bubble radius is a function of the circulation in a static environment and the saturation vapor pressure.

(b) Secondly, based on the relationships between a single bubble and bubble cluster proposed by Kubota et al. [32], the modelling of the condensation and vaporisation terms is given as follows. Neglecting the second-order terms and the surface tension, the Rayleigh–Plesset equations that describe the growth of a vapor bubble in a liquid the equation can reduce to:

$$\frac{dr_0}{dt} = \sqrt{\frac{2}{3}\frac{p_\infty - p_v}{\rho_l}}. \tag{3}$$

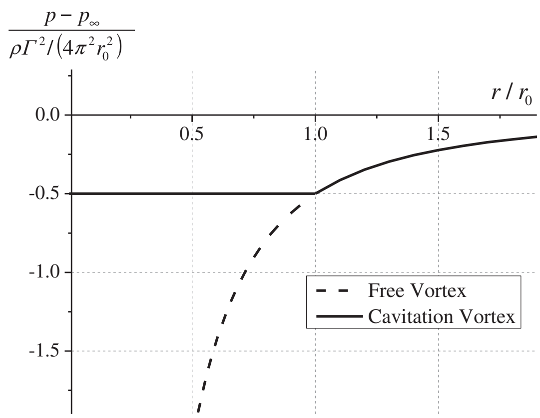

Fig. 1. Comparisons of the pressure distributions of free and cavitation vortices.