International Communications in Heat and Mass Transfer 78 (2016) 54–59

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

A new approach to solve the radiative transfer equation in plane-parallel semitransparent media with variable refractive index based on the discrete transfer method☆

S.M. Hosseini Sarvari

Mechanical Engineering Department, Shahid Bahonar University of Kerman, 76175-133 Kerman, Iran

ARTICLE INFO

Available online 5 September 2016

Keywords:
Radiation
Variable refractive index
Semitransparent media
Discrete transfer method

ABSTRACT

The radiative transfer equation in plane-parallel semitransparent media with variable refractive index is solved by the discrete transfer method. The medium refractive index is assumed to be constant in each control volume, such that the rays travel straight lines in control volumes, and redirect at interfaces. The effects of medium's refractive index on the curvature of solid angles are considered by a new set of weights, which are dependent on both ray path and medium's refractive index. The results are verified by comparing with a benchmark problem and the performance of the method is examined by various examples.

1. Introduction

Radiative transfer in semitransparent media (STM) has many applications in protecting coating, waveguide materials, optical measurement of flames, glass production and so on. Because of variable refractive index in STM, the rays propagate on curved lines. Hence, the analysis of radiative heat transfer in STM is more complex than that in gaseous media in which the radiative rays travels on straight lines.

In recent years, much attention has been paid to radiative heat transfer in STM. One of the earliest techniques for solving the RTE in STM has been developed by Siegel and Spuckler [1,2] to model a plane parallel composite medium, by dividing it into several slabs at uniform refractive index bounded by diffuse surfaces. Many ray tracing techniques have been developed to solve the RTE in STM with variable refractive index [3–13]. The curved ray tracing is very difficult, and hence, the application of these methods is mainly limited to one-dimensional problems. To overcome the difficulty due to curved ray tracing, many approaches have been developed to solve the RTE by other techniques such as discrete ordinate method [14–16], finite volume method [17,18], finite element method [19–23], and meshless method [24,25]. One of the key aspects of these methods is to transform the original RTE along the pencil of rays into appropriate form to involve the spatial derivatives. In addition, in almost all the above-mentioned methods, the formulations are not intuitive.

The discrete transfer method (DTM) has been known as a powerful, straightforward, and intuitive method for solving the RTE in participating media. The DTM developed by Shah and Lockwood [26], and improved by many researchers during past decades [27–31]. Recently, Krishna and Mishra [32] have extended the DTM to solve the RTE in STM with uniform non-unity and non-uniform linear refractive index distributions. The most challenging problem for solving the RTE by the DTM in STM with variable refractive index is to consider the curvature of solid angles due to the curvature of ray paths. Although they have considered the curvature of ray paths through the Snell's law of refraction [33], but they have not proposed an efficient approach to consider the effect of the curvature on solid angles. Instead, a unified set of constant weights in each direction have been proposed for solving the RTE in STM, with uniform and variable refractive index distributions, coincide to weights presented in their earlier work on participating media with unit refractive index [31]. Although this unified set of weights may lead to correct results for slight-sloped linear distribution of refractive index (as presented in [32]), but the results are more deviated from exact solution for hard-sloped linear and complex arbitrary profiles. Therefore, in the present work, a new set of weights is presented to include the effect of solid angle curvatures on the solution of the RTE. The RTE in STM is written based on a new quantity, namely the *refracted intensity*. The results of the present method are compared by a benchmark problem and its performance is examined by several examples.

2. Formulation

The mathematical formulation of RTE in an absorbing-emitting STM with variable refractive index, along the pencil of rays is as follows:

$$\frac{d\mathfrak{I}^{\pm}}{ds} + \kappa\,\mathfrak{I}^{\pm} = \kappa\,I_b \quad , \quad 0 \le \pm\,\theta^{\pm} \le \pi/2 \tag{1}$$

☆ Communicated by W.J. Minkowycz.
E-mail address: sarvari@uk.ac.ir.

http://dx.doi.org/10.1016/j.icheatmasstransfer.2016.08.017
0735-1933/

Nomenclature

I	radiative intensity, W/m^{2}sr
J	number of solid angles
K	number of volume elements
n	refractive index
Q	radiative heat flux, W/m^2
R	wall refractive ratio
s	geometric path length, m
T	temperature, K
x,y	Cartesian coordinates
Y	distance between parallel plates, m

Greek symbols

ε	emissivity
Φ	non-dimensional emissive power
φ	azimuthal angle, *rad*
k	absorption coefficient, m^{-1}
θ	polar angle, *rad*
Θ	refracted heat flux, W/m^2
σ	Stefan-Boltzmann constant, W/m^{2}K^4
τ	optical depth
ω	weight
Ψ	non-dimensional heat flux
$\Im$	refracted intensity, W/m^{2}sr
$\Re$	local refractive ratio

Subscripts

b	blackbody
w	wall

Superscripts

$+,-$	into positive and negative direction

Here, $\Im = I/n^2$ is called the *refracted intensity*. The superscripts − and + denote the irradiation rays received by the bottom and top wall surfaces, respectively. $I_b = \sigma T^4/\pi$ is the blackbody intensity, and κ is the absorption coefficient. The boundary condition for gray-diffuse wall surfaces is given by

$$\Im_{w^\mp}^{\pm} = \varepsilon_{w^\mp} I_{bw^\mp} + \frac{(1-\varepsilon_{w^\mp})}{\pi}\int_{\varphi=0}^{2\pi}\int_{\theta^\mp}\Im_{w^\mp}^{\mp}\cos\theta\,\sin\theta\,d\theta\,d\varphi \qquad (2)$$

where subscript $w^\mp$ denotes the value at bottom/top wall surface.

In order to solve the RTE by the DTM, the physical medium between parallel plates is divided into control volumes. The blackbody intensity and the radiative properties of the medium are taken as constant in each control volume. The refracted intensities may show a directional dependence on polar angle but not on azimuthal angle. Hence, the hemispheres on both wall surfaces are divided into annulus elements with equal polar angles, $\Delta\theta_{j,w}$ (see Fig. 1a).

The path of each irradiation ray must be traced back through the domain, being attenuated by the medium absorption and augmented by gas emission. However, for the medium with variable refractive index, the paths of rays are not straight lines. Hence, the medium refractive index is assumed to be constant in each control volume, but changes at its interfaces, in such a way that a ray travels a straight line in each control volume, and redirects at interface (see Fig. 1b).

The ray fired from bottom/top surface is traced along the control volume until it hits the interface, where the ray is redirected based on the Snell's law of refraction [33]

$$\sin\theta_{j,k}^{\pm} = \Re_k^{\pm}\sin\theta_{j,w^\pm} \qquad (3)$$

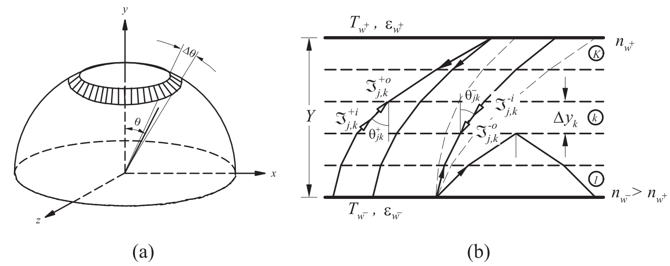

Fig. 1. (a) The hemisphere of solid angles on wall surfaces, and annular element with polar angle $\Delta\theta_{j,w}$, (b) The schematic of a plane-parallel STM with variable refractive index.

Here, $\Re_k^{\pm} = (n_{w^\pm}/n_k)$, called the *local refractive ratio*, shows the curvature of the ray path along the control volume k, with respect to the path of ray originated from top/bottom wall surface. The ray specularly reflects at that interface for which $\sin\theta_{j,k} > 1$. The ray tracing is terminated when the ray hits any boundary surface (Fig. 1b).

Integrating Eq. (1) over the geometric path length along the control volume k, leads to the following recursive relation:

$$\Im_{j,k}^{\pm o} = \Im_{j,k}^{\pm i} e^{-\kappa_k \Delta s_{j,k}^{\pm}} + I_{bk}\left(1 - e^{-\kappa_k \Delta s_{j,k}^{\pm}}\right), \quad j = 1,\ldots,J,\ k = 1,\ldots,K \qquad (4)$$

where index j and k runs over irradiation rays and control volumes, respectively. In Eq. (4), superscripts i and o denote the irradiation ray at the entering to and leaving from the control volume, respectively, and

$$\Delta s_{j,k}^{\pm} = \Delta y_k / \cos\theta_{j,k}^{\pm} \qquad (5)$$

is the distance traveled by the irradiation ray j through the control volume k. The refracted intensity emanating along the pencil of ray may be assumed to be constant in the solid angle. Therefore, the integral term in Eq. (2) can be replaced by a summation over solid angles as follows

$$\Im_{j,w^\mp}^{\pm} = \varepsilon_{w^\mp} I_{bw^\mp} + \frac{(1-\varepsilon_{w^\mp})}{\pi}\sum_{j=1}^{J}\Im_{j,w^\mp}^{\mp i}\left|\omega_{j,w^\mp}^{\mp}\right| \qquad (6)$$

where $\omega^{\pm}$ is the weight associated with direction j, along the rays received by top/bottom surface.

The weights are calculated by integration over the elemental polar angle as

$$\omega_{j,k}^{\pm} = \int_{\varphi=0}^{2\pi}\int_{(\theta_{j,k}^{\pm}-\Delta\theta_{j,k}^{\pm}/2)}^{(\theta_{j,k}^{\pm}+\Delta\theta_{j,k}^{\pm}/2)}\cos\theta_{j,k}\sin\theta_{j,k}\,d\theta_{j,k}\,d\varphi \qquad (7)$$

which resulted as follows

$$\omega_{j,k}^{\pm} = 2\pi\left[\sin^2\left(\theta_{j,k}^{\pm} + \Delta\theta_{j,k}^{\pm}/2\right) - \sin^2\left(\theta_{j,k}^{\pm} - \Delta\theta_{j,k}^{\pm}/2\right)\right] \qquad (8)$$

Then, by imposing the Snell's law, Eq. (3), into Eq. (8),

$$\omega_{j,k}^{\pm} = 2\pi\left(\Re_k^{\pm}\right)^2\left[\sin^2\left(\theta_{j,w^\pm} + \Delta\theta_{j,w^\pm}/2\right) - \sin^2\left(\theta_{j,w^\pm} - \Delta\theta_{j,w^\pm}/2\right)\right] \qquad (9)$$

and expanding the terms by trigonometric relations, leads to

$$\omega_{j,k}^{\pm} = 2\pi\left(\Re_k^{\pm}\right)^2\cos\theta_{j,w^\pm}\sin\theta_{j,w^\pm}\sin\Delta\theta_{j,w^\pm} \qquad (10)$$

The weights defined by Eq. (10) are not only dependent on polar angles originated from wall surfaces, but also dependent on the medium refractive index. In fact, the factor $(\Re_k^{\pm})^2$ shows the effect of medium's