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A structure-based model for transport in stably stratified homogeneous turbulent flows

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ABSTRACT

We present an extension that allows a recently proposed structure-based model for turbulent scalar transport to account for buoyancy effects. The proposed model is based on a generalization of the Interactive Particle Representation Model (IPRM) and is accompanied by a four-equation transport model that provides the turbulence scales needed for the closure of the complete structure-based model (SBM). The structure tensors and their invariants are used to model the additional buoyancy terms that emerge in the four-equation transport equations. Model parameters are set by matching the asymptotic decay exponents in decaying turbulence. The validity of the model is considered for a large number of different types of stably stratified flows at different Richardson numbers (Ri), showing encouraging results. The complete structure-based model achieves fair agreement with LES and DNS predictions for vertical shear in the presence of vertical mean stratification, while the structure tensors are shown to be suitable for use as diagnostic tools for the morphology of highly anisotropic turbulent structures. Additionally, the proposed model is shown to be sensitive to the variation of the inclination angle θ between the direction of the mean velocity gradient and the orientation of the mean scalar gradient. Furthermore, the model correctly predicts that the evolution of the inverse shear parameter is insensitive to the choice of inclination angle, yielding a turbulent Prandtl number close to unity, in accordance with DNS results.

1. Introduction

Turbulent transport in a stratified medium occurs in many engineering and geophysical applications, for example in atmospheric and marine flows. The importance of buoyant flows in technology and in nature has motivated many researchers to undetake their systematic study. An important parameter in the study of buoyant flows is the gradient Richardson number $Ri = \frac{N^2}{S^2}$, where N is the Brunt-Vaisala frequency, and $S = \sqrt{2S_{ij}S_{ij}}$ is the mean shear rate, where S_{ij} is the mean strain rate tensor. This parameter represents the ratio between the squares of the turbulence scales and measures the relative strength of mean shear and stratification effects and plays a significant role on the evolution of the turbulent field. For negative values of the square of the Brunt-Vaisala frequency N^2, the gradient Richardson number Ri becomes negative. This configuration is referred to as unstable stratification and results in the enhancement of the turbulent mixing caused by buoyancy in the ensuing convective motion. On the other hand, positive values of N^2 lead to a damping of the vertical motions due to buoyancy, a situation referred to as stable stratification.

Stable stratification is of particular scientific interest because it is associated with the strong anisotropy of the turbulent field caused by the combined action of mean shear and stratification. A number of experimental and numerical works on vertically stratified and vertically sheared flows has been performed, for example Holt et al. (1992), Piccirillo and Van Atta (1997), Itsweire et al. (1993), Sarkar (2003) and Schumann and Gertz (1995). For these cases, stratification tends to suppress vertical velocity fluctuations, while the mean shear is responsible for extracting energy from the mean field. For example, in the case where a streamwise velocity component with vertical gradient is present, the extracted energy is transferred preferentially to the streamwise direction, thus enhancing the streamwise fluctuations. At low values of Ri, shear effects dominate and the turbulent kinetic energy grows in time. However, the damping of the vertical motions is enhanced with increasing gradient Richardson number Ri, and above a critical value Ri_{cr}, the turbulent kinetic energy eventually decays. In the high Ri limit, the flow tends towards a two-component state composed of highly anisotropic structures. Often called pancakes, these struc-

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tures represent a two-component limit that is presumed to be approached but never actually reached in real flows (Rohr et al., 1988; Gerz et al., 1989).

Work on nonvertical stratification has also gained attention during the last decades, due to its relevance to engineering and environmental applications, such as coastal river outflows. The first systematic study was published by Jacobitz and Sarkar (1998) who showed that flow dynamics are controlled not only by the magnitude of the mean shear, but also by its orientation relative to the direction of gravity and stratification. In particular, orientation affects the growth rate of the turbulent kinetic energy, while it can push the critical gradient Richardson number to higher values.

The highly anisotropic nature of this type of flows is related to the occurrence of complex phenomena, such as strong pressure fluctuations or counter-gradient diffusion that are believed to be associated with large-scale distinct coherent structures (Kaltenbach et al., 1994; Hanjalic, 2002). All these phenomena are proven to constitute a particular challenge to conventional closure models that lack structural information (morphology of turbulence eddies) and thus exhibit a limited behavior, at best capturing only some of the underlying phenomena.

The significant effect that the large-scale structure has on the evolution of stratified flows motivated us to construct a structure-based model for scalar transport that accounts for buoyancy effects. For this purpose, in the current work we have extended the Structure-Based Model (SBM) for passive scalar transport that was recently proposed by Panagiotou and Kassinos (2016) based on the Interacting Particle Representation Model (IPRM) for homogeneous turbulence.

Here, we present the extended model and an evaluation of its performance for several cases involving different configurations of mean shear and buoyancy effects. The governing equations for the velocity and scalar fields are introduced in Section 2. The definitions of the turbulence structure tensors are briefly recounted in Section 3, where we also introduce the equivalent representation of the basic evolution equations in particle space. The extended IPRM is derived by modifying the basic transport equations in the particle space. In Section 4, we introduce a set of model equations for the turbulence scales that are sensitized to buoyancy effects and this allows us to bring closure to the extended IPRM model. In Section 5 the complete model is validated for a large number of test cases, yielding promising results. A summary and conclusions are given in Section 6.

2. Mathematical background

2.1. The governing equations

In the case of thermal stratification, a strong coupling between the velocity and scalar fields emerges. This coupling is described by the following set of governing equations for the velocity field u_i, the pressure field p and the scalar field ϕ:

$$u_{i,i} = 0\,, \tag{1a}$$

$$\partial_t u_i + u_j u_{i,j} = -\frac{p_{,i}}{\rho} + \nu u_{i,jj} - \beta\phi\hat{g}_i\,, \tag{1b}$$

$$\partial_t \phi + u_j \phi_{,j} = \Gamma\phi_{,jj}\,, \tag{1c}$$

where ρ is the density of the fluid, ν and Γ are the fluid viscosity and scalar diffusivity respectively and $\hat{g}_i$ is the gravitational field. In the case where ϕ represents the temperature field, β equals the thermal expansion coefficient, denoted as β_T

$$\beta = \beta_T = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial\phi}\right)_p, \tag{2}$$

where the subscript p signifies that the partial derivative is taken at constant pressure. For the case where ϕ represents the density field, β is defined by

$$\beta = -1/\bar{\Phi}_o\,, \tag{3}$$

where $\bar{\Phi}_o$ is the mean density field.

Hereafter, we adopt index notation, whereby repeated indexes imply summation and an index following a comma denotes differentiation with respect to the corresponding spatial coordinate. We consider turbulent flows, thus it is convenient to apply Reynolds' decomposition (Reynolds, 1895), expressing flow variables as a sum of a mean part, denoted by an overbar, and a fluctuating part, denoted by a prime,

$$u_i = \bar{u}_i + u'_i, \qquad p = \overline{p} + p', \qquad \phi = \overline{\phi} + \phi'\,. \tag{4}$$

Decomposition of Eq. (1) based on Reynolds' scheme yields an equivalent set of equations for the mean and fluctuating flow variables. In the homogeneous limit, transport equations for velocity and scalar fluctuations are given by

$$u'_{i,i} = 0\,, \tag{5a}$$

$$\partial_t u'_i + \bar{u}_j u'_{i,j} = -G_{ij}u'_j - u'_j u'_{i,j} - \frac{p'_{,i}}{\rho} + \nu u'_{i,jj} - \beta\phi'\hat{g}_i\,, \tag{5b}$$

$$\partial_t \phi' + \bar{u}_j \phi'_{,j} = -\Lambda_j u'_j - u'_j\phi'_{,j} + \Gamma\phi'_{,jj}\,, \tag{5c}$$

where $G_{ik} = \bar{u}_{i,k}$ and $\Lambda_k = \bar{\phi}_{,k}$ are the mean velocity gradient tensor and mean scalar gradient vector respectively.

3. An Interacting Particle Representation Model (IPRM) for active scalar transport.

In the mid-1990s, Kassinos and Reynolds (1994; 1995) introduced the idea that the componentality information contained in the Reynolds stress tensor provides an incomplete description of the turbulence anisotropy state. Hence, they introduced a set of tensors that characterize the anisotropy of the flow by providing a complete tensorial representation of turbulence structure. Examples of such tensors are the componentality (Reynolds stress) tensor, which describes the direction in which the fluctuations are more intense, and the dimensionality tensor, which gives information about the spatial extent of the coherent structures. The turbulence circulicity tensor provides information about the direction in which large-scale circulation in the turbulence is organized. A detailed description of the complete set of the turbulence structure tensors is given in several works, such as Kassinos et al. (2001), Stylianou et al. (2015) and Panagiotou and Kassinos (2016). The use of these tensors in the development of turbulence models that are sensitized to the morphology of the large-scale eddies is considered in Kassinos and Reynolds (1996; 1999), Poroseva et al. (2002) and Panagiotou and Kassinos (2016). For the development of structure-based turbulence closures, one needs to have access to the structure tensors for a range of canonical flows. The structure tensors can be obtained via Direct Numerical Simulations (DNS), and in the case of homogeneous turbulence, from spectral or pseudo-spectral direct numerical simulations. However, this approach is computationally costly and an overkill if all that one is interested in is to extract the one-point statistics of the turbulence. This motivated Kassinos and Reynolds (1994; 1995) to develop the Particle Representation Model (PRM), a numerically efficient and affordable method for carrying out exact Rapid Distortion Theory (RDT) computations. The PRM is a reduced representation that retains only a portion of the information that is normally carried in spectral simulations, but nevertheless provides access

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