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An improved anisotropy-resolving subgrid-scale model for flows in laminar–turbulent transition region

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ABSTRACT

Some types of mixed subgrid-scale (SGS) models combining an isotropic eddy-viscosity model and a scale-similarity model can be used to effectively improve the accuracy of large eddy simulation (LES) in predicting wall turbulence. Abe (2013) has recently proposed a stabilized mixed model that maintains its computational stability through a unique procedure that prevents the energy transfer between the grid-scale (GS) and SGS components induced by the scale-similarity term. At the same time, since this model can successfully predict the anisotropy of the SGS stress, the predictive performance, particularly at coarse grid resolutions, is remarkably improved in comparison with other mixed models. However, since the stabilized anisotropy-resolving SGS model includes a transport equation of the SGS turbulence energy, k_{SGS}, containing a production term proportional to the square root of k_{SGS}, its applicability to flows with both laminar and turbulent regions is not so high. This is because such a production term causes k_{SGS} to self-reproduce. Consequently, the laminar–turbulent transition region predicted by this model depends on the inflow or initial condition of k_{SGS}. To resolve these issues, in the present study, the mixed-timescale (MTS) SGS model proposed by Inagaki et al. (2005) is introduced into the stabilized mixed model as the isotropic eddy-viscosity part and the production term in the k_{SGS} transport equation. In the MTS model, the SGS turbulence energy, k_{es}, estimated by filtering the instantaneous flow field is used. Since the k_{es} approaches zero by itself in the laminar flow region, the self-reproduction property brought about by using the conventional k_{SGS} transport equation model is eliminated in this modified model. Therefore, this modification is expected to enhance the applicability of the model to flows with both laminar and turbulent regions. The model performance is tested in plane channel flows with different Reynolds numbers and in a backward-facing step flow. The results demonstrate that the proposed model successfully predicts a parabolic velocity profile under laminar flow conditions and reduces the dependence on the grid resolution to the same degree as the unmodified model by Abe (2013) for turbulent flow conditions. Moreover, it is shown that the present model is effective at transitional Reynolds numbers. Furthermore, the present model successfully provides accurate results for the backward-facing step flow with various grid resolutions. Thus, the proposed model is considered to be a refined anisotropy-resolving SGS model applicable to laminar, transitional, and turbulent flows.

1. Introduction

The main weakness of large eddy simulation (LES) is that a high grid density is required near the wall, especially in the streamwise and spanwise directions, relative to that for the simulation of the Reynolds-averaged Navier–Stokes equation (RANS). This high grid density greatly increases the computational cost at high Reynolds numbers. Meanwhile, it is also well known that some types of mixed subgrid-scale (SGS) models combining an isotropic eddy-viscosity model and a scale-similarity model are effective in reducing the grid resolution required to maintain a high prediction accuracy. Concerning scale-similarity model, the Bardina model (Bardina et al., 1983) is the most representative one. This model has been proved to remarkably improve the correlation between the modeled and actual SGS stresses in many *a priori* tests. However, this model provides insufficient mean energy transfer from the grid-scale (GS) field to the SGS field, and thus it must be adopted together with an eddy-viscosity model like the Smagorinsky model, i.e., a mixed model. Such a mixed model usually yields better results than eddy-viscosity models. Moreover, to

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enhance the effectiveness of the mixed model, Salvetti and Banerjee (1995) proposed a dynamic two-parameter mixed model, in which the model coefficient of the Bardina model term in the mixed model is defined as a variable and is determined automatically using the dynamic procedure by Germano et al. (1991) as well as the model parameter for the Smagorinsky model term. Furthermore, Horiuti (1997) improved the dynamic two-parameter mixed model by introducing a new scale-similarity term that models the modified SGS Reynolds stress, where the model coefficient of the new term is also computed dynamically. In the dynamic procedure, averaging in homogeneous directions is preferable in calculating the model parameters to avoid numerical instability. However, in engineering applications, there is generally no homogeneous direction, and thus the model parameters calculated at each grid point without averaging is used. In such cases, the obtained model parameter often causes the computation to be unstable, which imposes the use of a smaller computational time step and reduces the prediction accuracy.

On the other hand, Abe (2013) has recently proposed a stabilized mixed model, which maintains computational stability by using a unique procedure to prevent the energy transfer between the GS and SGS components induced by the scale-similarity term. At the same time, the transport equation of SGS turbulence energy, k_{SGS}, is solved, which is used not only to model the SGS eddy viscosity but also to scale the magnitude of the scale-similarity part. Since the model successfully predicts the anisotropy of the SGS stress, the predictive performance, particularly at coarse grid resolutions, is remarkably improved. Additionally, Abe and Ohtsuka (2014) has reported that the turbulent vortex motions enhanced by the scale-similarity term probably contributes to the improvement of the prediction accuracy.

However, the production term of the k_{SGS} transport equation employed in the stabilized anisotropy-resolving SGS model is proportional to the square root of k_{SGS}, which causes it to self-reproduce, and to the grid-filter width that is not related to the physical phenomena. Therefore, its applicability to flows accompanying both laminar and turbulent regions, which are commonly encountered in engineering applications, is considered not so high. For example, the laminar–turbulent transition predicted by the model probably depends on the inflow condition of k_{SGS} and the employed grid resolution. In addition, the definition of the grid-filter width used in the model differs significantly from the conventional width used in LES, which may reduce of the prediction accuracy in further applications to multi-physics simulations, e.g., flows with heat and mass transfer and flows including combustion or chemical reactions.

To resolve these issues, in the present study, the mixed-timescale (MTS) SGS model proposed by Inagaki et al. (2005) is introduced into the stabilized anisotropy-resolving SGS model as the isotropic eddy-viscosity part and the production term in the transport equation of k_{SGS}. In the MTS model, the SGS turbulence energy, k_{es}, estimated by filtering the instantaneous flow field is used. Since the k_{es} approaches zero by itself in the laminar flow region, as demonstrated by Inagaki et al. (2005), the self-reproduction property brought about by using the conventional k_{SGS} transport equation model could be eliminated. Thus, the present modification is expected to enhance the applicability of the model to flows in the laminar–turbulent transition region (Makino et al., 2015). The use of mixed timescale extends the applicability of the model to a variety of flow fields, as described by Inagaki et al. (2005); 2010). Moreover, it was confirmed that both the prediction accuracy and the computational stability of the MTS model are higher than those of the dynamic Smagorinsky (DS) model proposed by Germano et al. (1991). Furthermore, Inagaki et al. (2012) have proposed an MTS model for the thermal field, which is an extended version of the MTS model for the velocity field. Since the thermal MTS model has been confirmed to be a refined SGS model reflecting the Prandtl number effect by introducing the timescale of the thermal field, it is expected that the present anisotropy-resolving SGS model could be easily extended into an SGS model for the thermal and scalar fields.

In the present study, to enable the use of the conventional definition of the grid-filter width instead of the original one in Abe (2013), the influence of the definition is also investigated. It is found that some modifications are necessary for the use of the conventional definition to obtain prediction accuracy comparable to that of the Abe's original model, which is described in Section 5.1. After these modifications are made, the MTS model is introduced into the modified model. The performance of the modified model is tested in plane channel flows with various grid resolutions, as described in Section 5.2. It is also applied to a backward-facing step flow in Section 5.3 for the validation in more complex flows. Finally, Section 5.4 discusses the effectiveness of the present model in the plane channel flows at transitional Reynolds numbers.

2. Governing equations and stabilized anisotropy-resolving subgrid-scale model

2.1. Governing equations

The basic equations are the filtered Navier–Stokes and continuity equations for an incompressible fluid, which are given as follows:

$$\frac{\partial \bar{u}_j}{\partial x_j} = 0, \tag{1}$$

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_j \bar{u}_i}{\partial x_j} + \frac{\partial \tau_{ij}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j}, \tag{2}$$

$$\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j, \tag{3}$$

where $\bar{(\ )}$ denotes the grid-filtering operator and τ_{ij} is the SGS stress, which must be modeled.

2.2. Stabilized anisotropy-resolving subgrid-scale model

The stabilized anisotropy-resolving SGS model proposed by Abe (2013), hereafter referred to as the AR-Abe model, is based on a mixed model that expresses the SGS stress as the following linear combination of an isotropic eddy-viscosity model and a scale-similarity model:

$$\tau^*_{ij} = -2\,\nu_t \bar{S}_{ij} + \frac{2k_{SGS}}{\tau'_{kk}}\left\{\tau'^*_{ij} - \left(-2\nu' \bar{S}_{ij}\right)\right\}, \tag{4}$$

$$\tau'_{ij} = \left\{\left(\bar{u}_i - \tilde{\bar{u}}_i\right)\left(\bar{u}_j - \tilde{\bar{u}}_j\right)\right\}, \tag{5}$$

$$\nu' = -\frac{\tau'^*_{ij}\bar{S}_{ij}}{2\bar{S}_{ij}\bar{S}_{ij}}\ , \quad \bar{S}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{u}_j}{\partial x_i} + \frac{\partial \bar{u}_i}{\partial x_j}\right), \tag{6}$$

where ν_t is the SGS eddy viscosity, k_{SGS} is the SGS turbulence energy, and $\tau^*_{ij} = \tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk}$. The $\tilde{(\ )}$ denotes the space filtering operator. The representative points of this model, which appears in the second term on the right–hand side of Eq. (4), are as follows:

- Using ν' defined by Eq. (6) causes the energy dissipation from the scale-similarity part to be zero.
- The magnitude of the scale-similarity part is scaled by $\left(2k_{SGS} / \tau'_{kk}\right)$.