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Localised dynamics of laminar pulsatile flow in a rectangular channel

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ABSTRACT

The exploitation of flow pulsation in low-Reynolds number micro/minichannel flows is a potentially useful technique for enhancing cooling of high power photonics and electronics devices. Although the mechanical and thermal problems are inextricably linked, decoupling of the local instantaneous parameters provides insight into underlying mechanisms. The current study performs complementary experimental and analytical analyses to verify novel representations of the pulsating channel flow solutions, which conveniently decompose hydrodynamic parameters into amplitude and phase values relative to a prescribed flow rate, for sinusoidally-pulsating flows of Womersley numbers $1.4 \leq Wo \leq 7.0$ and a fixed ratio of oscillating flow rate amplitude to steady flow rate equal to 0.9. To the best of the authors' knowledge, the velocity measurements – taken using particle image velocimetry – constitute the first experimental verification of theory over two dimensions of a rectangular channel. Furthermore, the wall shear stress measurements add to the very limited number of studies that exist for any vessel geometry. The amplification of the modulation component of wall shear stress relative to a steady flow (with flow rate equal to the amplitude of the oscillating flow rate) is an important thermal indicator that may be coupled with future heat transfer measurements. The positive half-cycle time- and space-averaged value is found to increase with frequency owing to growing phase delays and higher amplitudes in the near-wall region of the velocity profiles. Furthermore, the local time-dependent amplification varies depending on the regime of unsteadiness: (i) For quasi-steady flows, the local values are similar during acceleration and deceleration though amplification is greater near the corners over the interval $0-0.5\pi$. (ii) At intermediate frequencies, local behaviour begins to differ during accelerating and decelerating periods and the interval of greater wall shear stress near the corners lengthens. (iii) Plug-like flows experience universally high amplifications, with wall shear stress greater near the corners for the majority of the positive half-cycle. The overall fluid mechanical performance of pulsating flow, measured by the ratio of bulk mean wall shear stress and pressure gradient amplifications, is found to reduce from an initial value of 0.97 at $Wo = 1.4$ to 0.28 at $Wo = 7.0$, demonstrating the increasing work input required to overcome inertia.

1. Introduction

1.1. Applications of pulsatile flow

Modern technological innovation in the ICT sector remains continually dependent on the removal of the significant amounts of heat generated by electronic and photonic components. In computing applications, the persistent size and cost reduction of semiconductor discretes has enabled the integration of hundreds of millions of transistors in three-dimensional architectures within microprocessors (Wilson, 2013). Single-phase liquid cooling systems involving micro- ($10\ \mu\text{m} < D_h \leq 200\ \mu\text{m}$) and mini- ($200\ \mu\text{m} < D_h \leq 3$ mm) channel heat sinks are commonly cited as a short-term solution to the high heat fluxes that result from sustained miniaturisation (Agostini et al., 2007). In telecommunications, the demand for global mobile connectivity imposed by online utilities such as cloud computing and the Internet of Things (IoT) has instigated a transition from copper to optics to transmit information. To further reduce the size of photonics packages, liquid-cooling in microchannels has been suggested as a component of the solution to remove this heat (Jeffers et al., 2014). While Poiseuille flow velocity profiles are self-similar – and therefore bound the Nusselt number – further enhancement may be accomplished through exploitation of flow unsteadiness, which is thought to alter the thickness of the hydrodynamic and thermal boundary layers, the near-wall gradients and the overall thermal resistance to heat transfer.

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Nomenclature

a, b	channel width, height [m]
D_h	hydraulic diameter [m]
f	oscillation frequency [Hz]
L_e	hydrodynamic entry length [m]
m, n	summation indices
p	pressure [Pa]
Q	flow rate [m^3/s]
Re	Reynolds number $(=\langle u\rangle D_h/\nu)$
Re_{δ_ν}	Re based on Stokes layer thickness $(=\langle u\rangle\delta_\nu/\nu)$
t	time [s]
u	velocity in the axial direction [m/s]
$\langle u\rangle$	space-averaged velocity [m/s]
Wo	Womersley number $(=\frac{1}{2}D_h\sqrt{\omega/\nu})$
x	axial flow coordinate [m]
y, z	coordinates normal to flow direction [m]

Greek symbols

β	function defined by Eq. (4b)
δ_ν	Stokes layer thickness [m]
η_τ	fluid mechanical performance coefficient
μ	viscosity [kg/(m·s)]
ν	kinematic viscosity [m^2/s]
ρ	density [kg/m^3]
τ	wall shear stress [Pa]
$\langle\tau\rangle$	space-averaged wall shear stress [Pa]
Φ	function defined by Eq. (4a)
ω	angular oscillation frequency [rad/s]
$\boldsymbol{\omega}$	vorticity [rad/s]

Subscripts

0	steady flow component
A	oscillating flow amplitude
y	acting about y axis
yx	x component with normal y
z	acting about z axis
zx	x component with normal z

Superscripts

$\prime$	phase relative to $\nabla p' = \nabla p_A \cdot \cos(\omega t)$
$\prime\prime$	phase relative to $Q'' = Q_A \cdot \sin(\omega t)$
$\prime\prime\prime$	phase relative to Q'', bulk mean phase removed

Experiments have demonstrated that significant heat transfer enhancement may be achieved, especially where bulk mean flow reversal occurs. Persoons et al. (2012) investigated the heat transfer enhancement of pulsating low-Reynolds number flow in a minichannel heat sink, for a range of modulation amplitudes and frequencies. While no discernible change was observed with the speed of oscillation, heat transfer was slightly diminished at low amplitudes, but enhanced by as much as 40% at high amplitudes. Gupta et al. (1982) discovered a qualitatively similar relationship with amplitude, measuring enhancement as high as 21%, though higher transfer rates were observed at lower frequencies. At the macro scale of industrial processes, Keil and Baird, 1971 demonstrated heat transfer enhancement of up to 31% compared to steady flow in a shell and tube heat exchanger. Without imposed space restrictions however, the low additional cost of a larger unit of sufficient capacity may limit adoption of the technology on the basis of thermal performance alone. More significant in such applications may be the capability of flow unsteadiness to mitigate fouling (Augustin and Bohnet, 1999), the global cost of which has been estimated at greater than \$4.13 billion in the petrochemical industry, due to increased capital expenditure, maintenance costs, and productivity losses (Van Nostrand et al., 1981). Fouling increases the thermal resistance, causes local hotspots by maldistribution of fluid in parallel channels, and demands additional pumping power because of the diminished cross-sectional area. The high near-wall velocity gradients of pulsating flow break up deposits at the heat exchanger surface, as demonstrated by Lelièvre et al. (2002) in the removal of bacteria from stainless steel equipment. Furthermore, utilisation of the thermal and mechanical effects of fluid flow, rather than the chemistry of cleaning agents, may reduce the toxicity and environmental impact of chemicals required in retrospective cleaning applications. Gillham et al. (2000) found that the cleaning rates of whey protein agglomerates, deposited in the thermal treatment of milk and dairy products, were especially sensitive to pulse amplitude and the presence of reverse flow. Enhanced cleaning and heat transfer rates of 250% and 100%, respectively, were measured using low frequency, large amplitude pulsations. Hence, the practical benefits of pulsatile flow depend on the input frequency and amplitude pulsation variables but, more specifically, on the influence of these parameters on hydrodynamic features of the flow such as near-wall velocity gradients and flow reversal.

1.2. Hydrodynamics of unsteady flow

Unsteady flow was treated mathematically as early as 1851 with Stokes' second problem (Stokes, 1851), which considers the one-dimensional shear flow of a viscous fluid near a flat plate oscillating in a direction parallel to its length. The solution models the spreading of transverse velocity oscillations from the boundary with normal velocity $\sqrt{2\nu\omega}$ and an oscillation amplitude that decays exponentially with displacement from the plate (Rosenhead, 1963). The region affected by oscillations is constrained since gradients in the flow are repeatedly annulled by those with opposite sign. The flow is inviscid far from the wall, outside of the so-called Stokes boundary layer where both viscosity and inertia must be considered. The behaviour of the flow is described by the relative size of the duct in comparison with the Stokes layer thickness, given by the Womersley number (Womersley, 1955):

$$Wo = \frac{D_h}{2}\sqrt{\frac{\omega}{\nu}} \tag{1}$$

where ω is the angular frequency. The linearity of the momentum equation imposes an inherent similarity between wall-driven fluid oscillations and those resulting from a harmonically-oscillating pressure gradient acting over the cross-sectional area of an enclosed vessel. Since the work of Sexl (1930), who solved the Navier–Stokes equations in a pipe geometry, a multitude of analytical studies have analysed the interdependence of the velocity, wall shear stress and pressure gradient with time and frequency, using mathematical solution techniques including Fourier expansion (Uchida, 1956), Laplace transform (Ito, 1953) and Green's function (Fan and Chao, 1965) methods. Ray et al. (2005) reorganised the 1-D pipe solution, based on the method of Uchida (1956), relative to a prescribed flow rate rather than pressure gradient. Haddad et al. (2010) expanded their work for a parallel plate channel, though using twice the hydraulic diameter of the pipe, and analysed the phenomenon of flow reversal in detail. Ohmi et al. (1981) defined boundaries to the quasi-steady, intermediate and inertia-dominated flow regimes based on adherence to certain mathematical approximations (Wo = 2.36 and 28 for pipe flow). The quasi-steady and inviscid flow assumptions are accurate where the amplitudes of the viscous and inertial terms constitute over 95% of that corresponding to the overall pressure gradient, respectively. Using this definition, transition between regimes in a rectangular channel has been investigated theoretically in a previous work by the current authors (Blythman et al., 2016b). Furthermore,