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Direct numerical simulation of a self-similar adverse pressure gradient turbulent boundary layer

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ABSTRACT

The statistical properties of a self-similar adverse pressure gradient (APG) turbulent boundary layer (TBL) are presented. The flow is generated via the direct numerical simulation of a TBL on a flat surface with a farfield boundary condition designed to apply the desired pressure gradient. The conditions for self-similarity and appropriate scaling are derived, with the mean profiles, Reynolds stress profiles, and turbulent kinetic energy budgets non-dimensionalised using this scaling. The APG TBL has a momentum thickness based Reynolds number range from $Re_{\delta_2} = 300$ to 6000, with a self-similar region spanning a Reynolds number range from $Re_{\delta_2} = 3500$ to 4800. Within this range the non-dimensional pressure gradient parameter $\beta = 1$. Two-point correlations of each of the velocity components in the streamwise/wall-normal plane are also presented, which illustrate the statistical imprint of the structures in this plane for the APG TBL.

1. Introduction

The efficient design and performance of many engineering systems rely on fluid flows remaining attached to aerodynamic surfaces in regions of adverse pressure gradient (APG). Separation of the boundary layer can potentially result in catastrophic consequences or at best sub-optimal performance. Adverse pressure gradients typically arise due to the presence of convex curved surfaces, such as those on wind turbine blades, turbo-machinery and aircraft wings. These configurations are difficult to systematically study, since the pressure gradient applied to the turbulent boundary layer (TBL) is constantly changing in the streamwise direction (Kitsios et al., 2011). There has been a long history of theoretical, experimental and numerical research into TBL. The vast majority of the research, however, has been centred on the zero pressure gradient (ZPG) case, while many aspects of turbulent structure and appropriate scaling of APG TBL remain largely unresolved. The study of APG turbulent boundary layers (TBL) in an appropriate canonical form is, therefore, of utmost importance to understand the influence of local pressure gradient.

The most appropriate canonical APG TBL to study is arguably one that is self-similar. A self-similar TBL (or portion thereof) is defined as one in which each of the terms in the governing equations have the same proportionality with streamwise position over the domain of interest (George and Castillo, 1993; Mellor and Gibson, 1966; Townsend, 1956). According to the definition in Mellor and Gibson (1966), this means that the non-dimensional pressure gradient, $\beta = \delta_1 (\partial_x P_e)/\tau_w$, must be constant, where $\partial_x P_e$ is the farfield pressure gradient, δ_1 is the displacement thickness, and τ_w is the mean shear stress at the well. Note this definition will be broadened in Section 3 of the present manuscript. For a ZPG TBL $\beta = 0$, for a favourable pressure gradient (FPG) $\beta < 0$, for an APG $\beta > 0$, and immediately prior to separation $\beta \to \infty$. Imagine two boundary layers, one starting with an APG that is then accelerated to ZPG, and another starting with a FPG that is then decelerated to ZPG. The statistical properties at the position of ZPG of these two scenarios are different from each other, and also different from the canonical ZPG flow (Perry et al., 2002). The flow structure, statistics, stability properties and scaling are all dependent upon the specific streamwise distribution of the pressure gradient. This illustrates the challenge of APG TBL studies and the value of studying the self-similar case.

Much of the theoretical work in the study of APG TBL is based on deriving the conditions and scaling properties for

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self-similar boundary layers, in which all statistics collapse down onto a single set of profiles for a given pressure gradient (Castillo and Wang, 2004; Durbin and Belcher, 1992; George and Castillo, 1993; Lighthill, 1963; Marusic and Perry, 1995; Mellor, 1966; Mellor and Gibson, 1966; Perry and Marusic, 1995; Townsend, 1956). Additional theoretical studies have focussed specifically on the limiting case of zero-shear-stress ($\beta \rightarrow \infty$) self-similar APG TBL, which is the scenario immediately prior to the point of mean separation (Chawla and Tennekes, 1973; Townsend, 1960). Attempts have also been made to collapse the statistical profiles of non-self-similar APG TBL using various definitions of the pertinent velocity and length scales (Maciel et al., 2006; Nickels, 2004; Zagarola and Smits, 1998).

There have been numerous experimental campaigns studying the effect of APG. Most of these studies have focussed on the statistical velocity profiles (Aubertine and Eaton, 2005; Cutler and Johnston, 1989; Elsberry et al., 2000; Monty et al., 2011; Simpson et al., 1977), with some recent measurements also presenting information on the spatial structure of such flows (Rahgozar and Maciel, 2011). A smaller number of experiments have also attempted to produce self-similar boundary layers, in which the statistical profiles at various streamwise positions collapse under the appropriate scaling Stratford (1959), Skåre and Krogstad (1994), and Atkinson et al. (2015a). The study of Skåre and Krogstad (1994) in particular focussed on the $\beta \rightarrow \infty$ case with a momentum thickness based Reynolds number, $Re_{\delta_2} = 4 \times 10^4$. The consistent observation across all of these studies, is the presence of a second outer peak in the variance of the velocity fluctuations located further away from the wall than the inner peak observed in ZPG TBL. This is due to the shear being distributed throughout the boundary layer imparted by the pressure gradient. This outer peak also becomes more prominent with increasing pressure gradient.

Direct numerical simulation (DNS) have also been undertaken of both self-similar and non-self-similar APG TBL. Each of the following DNS are performed in a rectangular domain, with the APG applied via the prescription of the farfield boundary condition. The first DNS of an APG TBL was that of the Spalart and Watmuff (1993), which produced a non-self-similar TBL with $Re_{\delta_2} = 1600$, and $\beta = 2$. There have also been several DNS of separated APG flows (Gungor et al., 2012; Na and Moin, 1998; Skote and Henningson, 2002), with the most recent of which (Gungor et al., 2012) having the largest Reynolds number of $Re_{\delta_2} = 2175$. The only attempted DNS of self-similar boundary layers are those of Skote et al. (1998) and Lee and Sung (2008). In the study of Skote et al. (1998) two DNS were presented, the first with Reynolds numbers ranging from $Re_{\delta_2} = 390$ to 620 with $\beta = 0.24$, and the second having a Reynolds number range of $Re_{\delta_2} = 430$ to 690 with $\beta = 0.65$. In the more recent study of Lee and Sung (2008) a higher Reynolds number APG TBL DNS was presented with $Re_{\delta_2} = 1200$ to 1400, and also with a stronger pressure gradient of $\beta = 1.68$.

The focus of the present study is to add to the current body of APG TBL DNS databases, and in particular address the need for higher Reynolds number self-similar APG flows. Specifically we present a DNS of an APG TBL with a Reynolds number range of $Re_{\delta_2} = 300$ to 6000, which is larger in both range and magnitude of the aforementioned APG TBL DNS studies. Self-similarity of the TBL is also demonstrated from $Re_{\delta_2} = 3500$ to 4800, within which $\beta = 1$. In the current manuscript we present the details of DNS, characterise the APG TBL on the basis of scaling properties, one-point and two-point statistics. Firstly in Section 2, an overview of the TBL DNS code is presented along with the farfield boundary condition (BC) required to generate the self-similar APG TBL. The APG TBL is characterised and compared to the ZPG TBL on the basis of standard boundary layer properties including integral length and velocity scales. In Section 3, the conditions for self-similarity (and associated scaling) are derived from the boundary layer equations and evaluated for both the APG and ZPG cases. Profiles of the mean velocity deficit and Reynolds stresses from the DNS of the APG are then compared to those of the ZPG DNS on the basis of both the derived scaling and also viscous scaling in Section 4. In Section 5 two-point correlations of each of the velocity components are presented in the streamwise/wall-normal plane for the APG TBL and contrasted with previous ZPG DNS results. Finally concluding remarks are made in Section 6.

2. Direct numerical simulation

The code adopted within solves the Navier–Stokes equations in a three-dimensional rectangular volume, with constant density (ρ) and kinematic viscosity (ν). The three flow directions are the streamwise (x), wall-normal (y) and spanwise (z), with instantaneous velocity components in these directions of U, V and W. Notation used for the derivative operators in these directions are $\partial_x \equiv \partial/\partial x$, $\partial_y \equiv \partial/\partial y$, and $\partial_z \equiv \partial/\partial z$. Throughout the paper the mean velocity components are represented by $(\langle U\rangle, \langle V\rangle, \langle W\rangle)$, with the averaging undertaken both in time and along the spanwise direction. The associated fluctuating velocity components are (u, v, w).

Details of the algorithmic approach to solve the equations of motion are as follows. A fractional-step method is used to solve the governing equations for the velocity and pressure (P) fields (Harlow and Welch, 1965; Perot, 1993). Fourier decomposition is used in the periodic spanwise direction, with compact finite difference in the aperiodic wall-normal and streamwise directions (Lele, 1992). The equations are stepped forward in time using a modified three sub-step Runge–Kutta scheme (Simens et al., 2009). The code utilises MPI and openMP parallelisation to decompose the domain. For further details on the code and parallelisation, the interested reader should refer to Borrell et al. (2013) and Sillero (2014). In the following sections we present: the boundary conditions necessary to implement the ZPG and APG TBL; numerical details; and characterise both the APG and ZPG TBL flows.

2.1. Boundary conditions

The boundary conditions of the ZPG TBL DNS code are outlined below. The bottom surface is a flat plate with a no-slip (zero velocity) BC. The spanwise boundaries are periodic. Due to the TBL growing in height as it develops in the streamwise direction, a downstream streamwise normal recycling plane is copied, and mapped to the inlet BC (Sillero et al., 2013). At the farfield boundary a zero spanwise vorticity condition is applied, and the wall normal velocity specified. It is important that the wall normal velocity be prescribed, as opposed to the streamwise velocity, so as to not over constrain the system (Rheinboldt, 1956). This may not be a significant problem for ZPG TBL, but becomes an increasingly significant issue as the pressure gradient increases. The wall normal velocity at this boundary is given by

$$V_{ZPG}(x) = U_{ZPG}\ \partial_x \delta_1(x), \quad (1)$$

where U_{ZPG} is the constant freestream streamwise velocity, and δ_1 is the displacement thickness (Sillero, 2014).

Due to the properties of the APG TBL, we also use a slightly different definition of displacement (δ_1) thickness, and of the momentum thickness (δ_2) for that matter. These length scales are given by

$$\delta_1(x) = \int_0^{\delta(x)} \left(1 - \frac{\langle U\rangle(x,y)}{U_e(x)}\right) dy, \text{ and} \quad (2)$$

$$\delta_2(x) = \int_0^{\delta(x)} \left(1 - \frac{\langle U\rangle(x,y)}{U_e(x)}\right) \frac{\langle U\rangle(x,y)}{U_e(x)}\ dy, \quad (3)$$