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Numerical investigation of a spatially developing turbulent natural convection boundary layer along a vertical heated plate

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ABSTRACT

Large-eddy simulation (LES) on a spatially developing natural convection boundary layer along a vertical heated plate was conducted. The heat transfer rate, friction velocity, mean velocity and temperature, and second-order turbulent properties both in the wall-normal and the stream-wise direction showed reasonable agreement with the findings of past experiments. The spectrum of velocity and temperature fluctuation showed a -2/3-power decay slope and -2-power decay slope respectively. Quadrant analysis revealed the inclination on Q1 and Q3 in the Reynolds stress and turbulent heat flux, changing their contribution along the distance from the plate surface. Following the convention, we defined the threshold region where the stream-wise mean velocity takes local maximum, the inner layer which is closer to the plate than the threshold region, the outer layer which is farther to the plate than the threshold region. The space correlation of stream-wise velocity tilted the head toward the wall in the propagating direction in the outer layer; on the other hand, the correlated motion had little inclination in the threshold region. The time history of the second invariant of gradient tensor Q revealed that the vortex strength oscillates both in the inner and the outer layers in between the laminar and the transition region. In the turbulent region, the vortex was often dominant in the outer layer. Instantaneous three-dimensional visualization of Q revealed the existence of high-speed fluid parcels associated with arch-shape vortices. These results were considered as an intrinsic structure in the outer layer, which is symmetrical to the structure of canonical smooth/rough wall bounded layer flow in forced convection.

1. Introduction

Turbulent natural convection boundary layers have been considered to be a significant phenomenon in heat transportation originating from enormous research and applicational background, such as heat transfer in industrial machinery, building heat transfer, and life time of glaciers in oceans. The characteristics of a spatially developing natural convection boundary layer along a vertical heated plate (NCBL) have been widely investigated through theories, experiments and numerical simulations using sophisticated techniques. Theoretical investigations have given us a profound understanding of several issues of NCBL, such as, scaling of near-wall transport (George and Capp, 1979; Hölling and Herwig, 2005; Wells and Worster, 2008) and the amplification of disturbance (Dring and Gebhart, 1968; Cheesewright, 1968) experimentally revealed the heat transfer rate including the laminar and turbulent region. The transition from laminar to turbulent flow was observed where Rayleigh number $\mathrm{Ra}_x = g\beta\Delta\Theta x^3/\nu\alpha$ (g: gravity acceleration, β: coefficient of thermal expansion, $\Delta\Theta$: temperature difference between the heated plate and ambient air, x: vertical distance from the heated lower edge, ν: kinematic viscosity, α: thermal diffusivity) is around 10^{9-10} for air. Tsuji and Nagano (1988a,1988b) measured the turbulence stochastic properties with high space and time resolution. Kitamura et al. (1985) suggested the existence of burst-like phenomena and an inflow-ejection cycle as is observed in forced convection (FC). Tsuji et al. (1992) discussed the structure of temperature from spatio-temporal correlation measurement. The reproducibility of flow characteristics by numerical simulation using the Reynolds Averaged Navier-Stokes equation model and the Reynolds stress model have also been investigated by To and Humphrey (1986) and Peeters and Henkes (1992) respectively. The usage of direct numerical simulations or large-eddy simulations (LESs) seems promising for reproducing NCBL; however, most simulation configurations are restricted in an internal-flow condition such as a tall cavity with a heated (cooled) wall (Versteegh and Nieuwstadt, 1999; Barhaghi and Davidson, 2007; Kizildag et al., 2014). Case studies taking into account spatial development have not been fertile (Abedin et al., 2009 for time-developing flow configuration with a periodic condition in the main stream direction, and Barhaghi et al., 2006 for a heated cylinder). Hence, there

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is still a need for clarification of turbulent characteristics. For example, the coherent structure, burst-like phenomena, and sweep-ejection cycle, are focused on only in the experiment mentioned above and uncertainties remain.

As the first step, our numerical study focuses if LES recreates spatial development of heat transfer, the mean profiles of momentum and temperature, and next, the vortex structure sustained in NCBL for a better understanding of turbulent characteristics. The simulations were carried out under several grid resolutions and viscosity conditions. The numerical set-up is explained in Section 2. The results of first- and second-order moments, spectrum of velocity and temperature, the vortex motion, joint probability density function of momentum and heat flux, and space correlation are shown and discussed in Section 3. In addition, an instantaneous three-dimensional vortex structure is visualized and its configuration is discussed. Finally, conclusions are presented in Section 4.

2. Methods and conditions

Solving the incompressible Navier-Stokes equation with the Boussinesq approximation enabled the numerical recreation of three-dimensional flow and temperature fields. Governing equations is written as:

$$\frac{\partial \bar{u}_i}{\partial t}+\frac{\partial \bar{u}_i\bar{u}_j}{\partial x_j}=-\frac{\partial \bar{P}}{\partial x_i}-\frac{\partial \tau_{ij}}{\partial x_j}+\nu\frac{\partial^2\bar{u}_i}{\partial x_j\partial x_j}+\delta_{i1}g\beta\bar{\theta}, \tag{1}$$

$$\frac{\partial \bar{\theta}}{\partial t}+\frac{\partial \bar{u}_j\bar{\theta}}{\partial x_j}=-\frac{\partial \pi_j}{\partial x_j}+\frac{\nu}{Pr}\frac{\partial^2\bar{\theta}}{\partial x_j\partial x_j}, \tag{2}$$

$$\frac{\partial \bar{u}_i}{\partial x_i}=0. \tag{3}$$

Here u_1, u_2, and u_3 denote upward, span-wise and wall-normal velocity respectively, θ the temperature difference from ambient air, and P the pressure divided by density. The suffix 1 corresponds to x- direction, 2 to y- direction, and 3 to z- direction in Figs. 1 and 2. Over-bar $\bar{x}$ denotes the filter operation, which will be omitted from the next paragraph for brevity. Pr is Prandtl number, set at 0.71 in all cases. τ_{ij} denotes subgrid-scale stress of momentum:

$$\tau_{ij}=-2C\Delta^2\left|\bar{S}\right|\bar{S}_{ij}, \tag{4}$$

and π_j is that of heat which was derived by the analogy of the heat diffusivity flux using constant subgrid-scale Prandtl number, 0.5 .We conducted Large-eddy simulation with the subgrid-scale model based on coherent structures (Coherent structure Smagorinsky model (Kobayashi, 2005)). The model evaluates local coefficients of the subgrid-scale stress, C in space and time. We followed original derivation of the subgrid-scale stress described as:

$$C=C_1|F_{CS}|^{3/2};\ \text{and}\, C_1=1/20, \tag{5}$$

where C_1 is the model constant, F_{CS} the coherent structure function defined as the second invariant normalized by the magnitude of a velocity gradient tensor:

$$F_{CS}=\frac{Q}{E}, \tag{6}$$

$$Q=\frac{1}{2}\left(\bar{W}_{ij}\bar{W}_{ij}-\bar{S}_{ij}\bar{S}_{ij}\right);\ \text{and}\, E=\frac{1}{2}\left(\bar{W}_{ij}\bar{W}_{ij}+\bar{S}_{ij}\bar{S}_{ij}\right), \tag{7}$$

where, $|\bar{S}|=(2\bar{S}_{ij}\bar{S}_{ij})^{1/2}$, $\bar{S}_{ij}=1/2(\partial\bar{u}_j/\partial x_i+\partial\bar{u}_i/\partial x_j)$ and $\bar{W}_{ij}=1/2(\partial\bar{u}_j/\partial x_i-\partial\bar{u}_i/\partial x_j)$. The model has shown comprehensive ability to capture the effective turbulent motion almost same as the dynamic models on the variety of flow, such as turbulent rotating channel flow (Kobayashi, 2005), Magneto-hydrodynamic flow (Kobayashi, 2008), and backward-facing step flow (Kobayashi et al., 2008). For a farther description of performance, see the references.

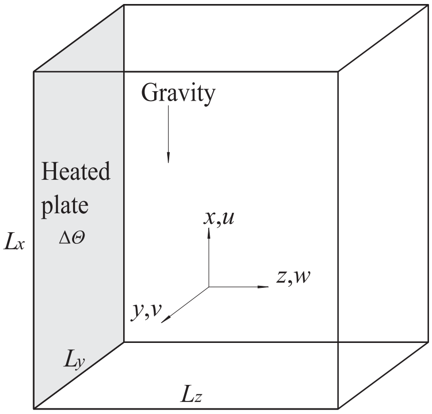

Fig. 1. Calculation coordinates.

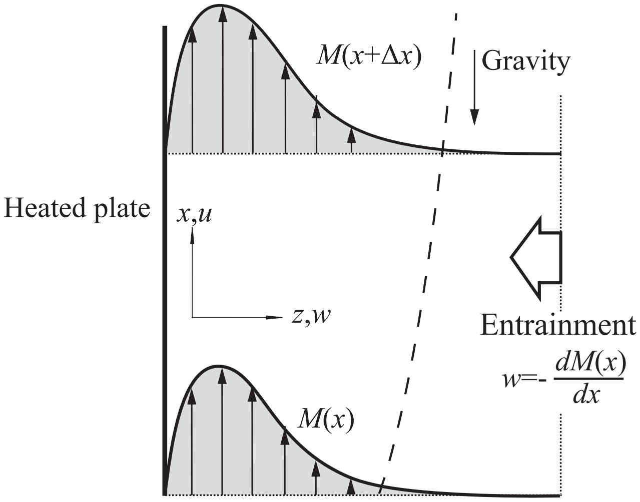

Fig. 2. Entrainment of developing boundary layer.

Space discretization was done by conservative second-order accurate central differentiation. Time development was performed using the Adams-Bashforth method both in advection and diffusion terms of the momentum and the heat. The boundary condition on the outlet upper plane was the convective boundary condition, where instantaneous negative velocity was prohibited. The lower plane was free-slip surface for the momentum and 0-gradient for the heat. Sidewalls were periodic boundary for the momentum and the heat. These boundary implementations allow mass inlet only from the counterpart to the heated plate. It is assumed that the boundary of the external flow should be same as an infinite distance (Craske and Reeuwijk, 2013). Hence, the boundary counterpart to the heated plate was assessed using the following diagnostic equations:

$$w=-\frac{dM}{dx}, \tag{8}$$

$$M=\int_0^{\infty}U_{tmp}\,dz, \tag{9}$$

$$U^{n+1}_{tmp}=\frac{\Delta t}{T}\left\langle u^{n+1}\right\rangle_y+\left(1-\frac{\Delta t}{T}\right)U^{n}_{tmp.} \tag{10}$$

Here, w denotes the wall-normal velocity, M the total mass flux passing the wall-normal direction at height x, and U_{tmp} the pro-