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Direct numerical simulation of a turbulent wake: The non-equilibrium dissipation law

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ABSTRACT

A Direct Numerical Simulation (DNS) study of an axisymmetric turbulent wake generated by a square plate placed normal to the incoming flow is presented. It is shown that the new axisymmetric turbulent wake scalings obtained recently for a fractal-like wake generator (Dairay et al., 2015), specifically a plate with irregular multiscale periphery placed normal to the incoming flow, are also present in an axisymmetric turbulent wake generated by a regular square plate. These new scalings are therefore not caused by the multiscale nature of the wake generator but have more general validity.

1. Introduction

Axisymmetric turbulent wakes have been extensively studied experimentally and numerically (see for example Johansson et al., 2003). A problem of particular interest remains however the prediction of the scaling laws for the wake's width δ and the centreline velocity deficit u_0 along the streamwise distance x. For the turbulent axisymmetric and self-preserving wake, these scalings laws can be derived from knowledge of the dissipation rate ϵ scalings (Townsend, 1976; George, 1989). In a recent study, Nedić et al. (2013) proposed an extension of the theory established in George (1989) assuming the non-equilibrium dissipation scaling (see Vassilicos, 2015, for details)

$$\epsilon = C_\epsilon \frac{K^{3/2}}{\delta} \quad \text{with } C_\epsilon \sim Re_G^m / Re_l^m \tag{1}$$

where K is the turbulent kinetic energy, Re_G is a global Reynolds number determined by the inlet conditions and Re_l is a local Reynolds number based on local velocity and length scales. This theory has recently been tested in detail and revised by Dairay et al. (2015). Invoking an assumption of constant anisotropy, Dairay et al. (2015) have shown that it is possible to derive scaling laws for u_0 and δ for any values of the exponent m in (1) (this assumption actually replaces the usual assumption of self-similarity of every single term of the turbulent kinetic energy equation which turns out to be incorrect for some of the terms). They obtain $\delta(x)/\theta = B((x-x_0)/\theta)^\beta$ and, $u_0(x)/U_\infty = A((x-x_0)/\theta)^\alpha$ where θ is the momentum thickness, U_∞ is the freestream velocity, x_0 is a virtual origin, $\beta = (1+m)/(3+m)$, $\alpha = -2\beta = -2(1+m)/(3+m)$, $B \sim (L_b/\theta)^{\frac{2m}{3+m}}$ and $A = B^{-2}$. In Dairay et al. (2015), these predictions have been found to be in agreement with both numerical and experimental data for an axisymmetric turbulent wake generated by an irregular plate. The aim of this paper is to use DNS data to interrogate the existence of the new non-equilibrium dissipation law (1) and its wake-law consequences in a more "conventional" turbulent wake generated by a square plate.

2. Flow configuration and numerical methods

In the present study, a turbulent wake is generated by a square plate of surface area A placed normal to the incoming flow (see Fig. 1 for illustration). The surface area of the square plate is the same as the one of the irregular plate used in Dairay et al. (2015). In the Cartesian coordinate system (O; x, y, z), the domain is $\Omega = [-x_p, L_x - x_p] \times [-L_y/2, L_y/2] \times [-L_z/2, L_z/2]$ where $x_p = 10L_b$ is the longitudinal location of the plate, the origin O is located at the centre of the plate and $L_x \times L_y \times L_z = 120L_b \times 15L_b \times 15L_b$ where $L_b = \sqrt{A}$ is the reference length of the flow (see Fig. 1). For the sake of simplicity the radial distance $r = \sqrt{y^2+z^2}$ and the polar angle $\varphi = \arctan(y/z)$ are also introduced hereinafter.

Mean quantities $< f >(x, r)$ of a field $f(x, r, \varphi, t)$ are estimated by averaging over time and over the homogeneous polar direction φ in the cylindrical coordinate system (x, r, φ). The mean streamwise velocity component $< u_x >(x, r)$ is denoted U. The momentum thickness θ is defined by $\theta^2 = (1/U_\infty^2)\int_0^\infty U_\infty(U_\infty - U)rdr = const.$ and the wake's width is here characterised by the integral wake's width δ, with $\delta^2(x) = (1/u_0)\int_0^\infty (U_\infty - U)rdr$ where $u_0(x) = U_\infty - U(x, r/L_b = 0)$ is the centreline velocity deficit. The

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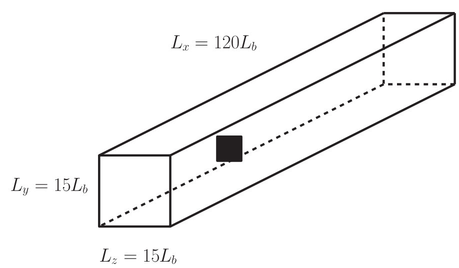

Fig. 1. Schematic view of the computational domain.

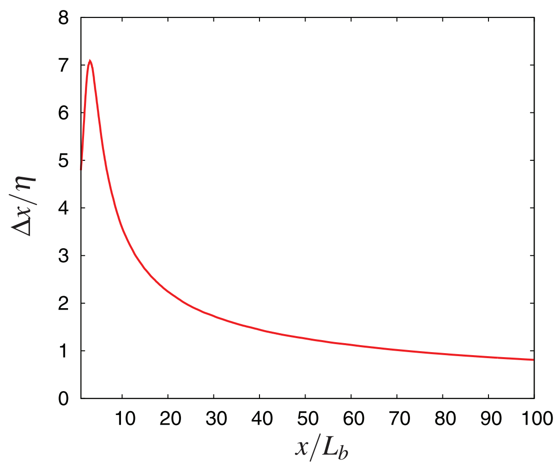

Fig. 2. Streamwise evolution of the ratio $\Delta x/\eta$. The Kolmogorov microscale η has been computed on the basis of the maximum value of $\varepsilon(x, r)$ along r.

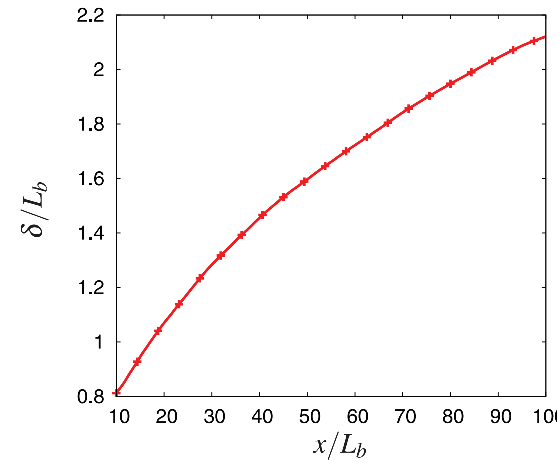

Fig. 3. Streamwise evolution of the wake width δ/L_b.

global Reynolds number Re_G based on the reference length L_b and the freestream velocity U_∞ is $Re_G = 5000$. The local Reynolds number Re_l is defined by $Re_l(x) = \sqrt{K_0(x)}\delta(x)/\nu$ where K_0 is the turbulent kinetic energy at a centreline location.

The finite difference code *Incompact3d* (Laizet and Lamballais, 2009; Laizet et al., 2010) is used to solve the incompressible Navier-Stokes equations. The modelling of the plate is performed by an Immersed Boundary Method, following a procedure proposed by Parnaudeau et al. (2008). Inflow/outflow boundary conditions are assumed in the streamwise direction with a uniform fluid velocity U_∞ without turbulence as inflow condition and a 1D convection equation as outflow condition. The boundary conditions in the two spanwise directions are periodic. The computational domain is discretized on a Cartesian grid of $n_x \times n_y \times n_z = 3841 \times 480 \times 480$ points. In terms of Kolmogorov microscale η, as illustrated in Fig. 2, the spatial resolution is at worst $\Delta x = \Delta y = \Delta z \approx 7\eta$ (where the turbulence is at its most intense) and at best $\Delta x = \Delta y = \Delta z \approx 0.8\eta$ (at the end of the computational domain where the turbulence has decayed). In the range $10 \le x/L_b \le 100$, which is the range of interest of our study, the spatial resolution is always below 4η. In a recent resolution study, Laizet et al. (2015) have shown that a spatial resolution of 7η or 5η is sufficient to reproduce experimental results with an error margin of about 10% or 5% respectively (for one-point first and second order statistics). They also showed that quantities such as the turbulence dissipation rate require a resolution of at least 4η to be well captured. For the spatial derivatives, sixth-order centred compact schemes (Lele, 1992) are used. To control the residual aliasing errors, a small amount of numerical dissipation is introduced only at scales very close to the grid cutoff. This very targeted regularization is ensured by the differentiation of the viscous term that is sixth-order accurate (Lamballais et al., 2011). The time integration is performed using an explicit third-order Adams-Bashforth scheme with a time step $\Delta t = 5 \times 10^{-3} L_b/U_\infty$ (corresponding to a CFL number of 0.16 and ensuring $\Delta t < 0.014\tau_\eta$ where τ_η is the Kolmogorov time-scale). Full details about the code "Incompact3d" can be found in Laizet and Lamballais (2009); Laizet et al. (2010); Laizet and Li (2011) (see also the link www.incompact3d.com).

The collection of data for the turbulent statistics is done over a time $T = 3850L_b/U_\infty$, corresponding to approximately 25 seconds of the experiments in Nedić et al. (2013) and to 423 cycles based on the Strouhal number $St = f_{vs}L_b/U_\infty = 0.11$ associated with the vortex shedding frequency f_{vs} (see Nedić et al., 2013). This time is also the same as the one used in Dairay et al. (2015) ensuring good convergence of the DNS statistics.

The streamwise evolution of δ is plotted in Fig. 3. At $x = 100L_b$ (the most distant streamwise location that we are considering in the present paper), $\delta \approx 2.12L_b$. This means that the domain half-width is $L_y/2 = L_z/2 = 7.5L_b \approx 3.54\delta$ at $x = 100L_b$. According to Redford et al. (2012), the critical value needed to ensure that the lateral boundary conditions do not affect the wake development is $L_z/2 = L_y/2 \approx 2.95\delta$. The lateral dimensions of our domain therefore appear sufficiently large to avoid any significant contamination from the lateral boundaries even at $x = 100L_b$.

The DNS data can first be used to assess the validity of the local isotropy assumption commonly used in the experimental framework. In Fig. 4 we compare $\varepsilon_{iso} = 15\nu\langle(\partial u'_x/\partial x)^2\rangle$ with the actual dissipation rate $\varepsilon_{full} = 2\nu\langle s_{ij}s_{ij}\rangle$ where $s_{ij} = (1/2)(\partial u'_i/\partial x_j + \partial u'_j/\partial x_i)$. It is clear from Fig. 4 (right) that $\varepsilon_{iso}/\varepsilon_{full}$ lies between 0.96 and 1.04 in the range $10 \le x/L_b \le 100$.

3. Axisymmetry of wake statistics

A quantitative evaluation of the statistical axisymmetry of the flow generated by the square plate can been obtained by computing the mean values of the coefficient of variance $c_v(x, r) \equiv 100\sqrt{(1/N_\varphi)\sum_\varphi (S(x, r, \varphi) - \langle S\rangle(x, r))^2}/\langle S\rangle(x, r)$ where N_φ is the number of polar angles and S stands for mean flow, turbulent kinetic energy or dissipation rate of turbulent kinetic energy. The streamwise variations of the radially averaged coefficient of variance $\overline{c_v}(x) \equiv (1/N_r)\sum_r c_v(x, r)$ are plotted in Fig. 5 while the irregular plate data of Dairay et al. (2015) are added for comparison. Fig. 5 shows that, at $x = 10L_b$, there is already less than 4% variation in all statistics demonstrating the good axisymmetry of the flow generated by the square plate at $x > 10L_b$.

4. Similarity of the axisymmetric turbulent wakes

The axisymmetry of the flow generated by the square plate has been carefully checked. The next step of the analysis is to investigate the similarity properties of mean flow statistics. Self-similar