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Growth of boundary-layer streaks due to free-stream turbulence

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ABSTRACT

The growth of laminar boundary-layer streaks caused by free-stream turbulence encountering a flat plate in zero-pressure-gradient conditions is investigated experimentally in a wind tunnel and numerically by solving the unsteady boundary-region equations. A comparative discussion amongst the most relevant theoretical frameworks, such as the Goldstein theory, the Taylor–Stewartson theory, the optimal-growth theory and the Orr-Sommerfeld theory, is first presented and parallels and complimentary aspects of the theories are pointed out to justify the use of the Goldstein theory in our study. The statistical properties of the positive and negative fluctuations of the laminar streaks are discussed, showing how the total time average of the boundary-layer fluctuations masks the true character of the disturbance flow and revealing that the maximum values and the root-mean-square of positive and negative fluctuations grow downstream at the same rate. The downstream growth rate of the low-frequency disturbances and the decay rate of the high-frequency disturbances are also computed for the first time. The numerical solutions of the unsteady boundary-region equations are compared successfully with the streak profiles measured in the wind tunnel and with direct numerical simulation results available in the literature.

1. Introduction

The topic of boundary-layer transition to turbulence is of extreme importance in the aeronautical, turbomachinery, and maritime industries because of the much higher wall friction drag exerted by the flow in the turbulent regime than in the laminar regime. It is therefore paramount for scientists and engineers to understand the physical causes of such a process, to predict its occurrence, and to control it to delay the breakdown of the flow to turbulence.

1.1. The laminar streaks and bypass transition

Boundary-layer transition in the presence of a low level of free-stream turbulence is widely acknowledged to be initiated by exponentially growing Tollmien–Schlichting (TS) waves (Schubauer and Skramstad, 1947), but for free-stream turbulence levels which are comparable to and larger than about 1%, the laminar-flow breakdown appears to be linked to streamwise-elongated regions of high and low streamwise velocity which dominate the boundary-layer core. These structures have been termed laminar streaks, breathing modes, or Klebanoff modes (Kendall, 1990; Klebanoff, 1971). The streaks are believed to be responsible for bypass transition in which the role of unstable TS waves, as predicted by the linear stability theory, is likely to be marginal or even irrelevant (Matsubara and Alfredsson, 2001). The breakdown of the Klebanoff modes is caused by secondary instability and occurs abruptly along the flat plate. In most industrial flow scenarios, free-stream flows impinging on rigid surfaces are common and bypass transition is the norm. This has spurred a growing interest in the Klebanoff modes over the last twenty years.

A complete understanding of the bypass transition has not been attained, despite research efforts based on experiments (Fransson et al., 2005; Hernon et al., 2007a, 2007b; Matsubara and Alfredsson, 2001; Nolan et al., 2010; Pook et al., 2016), numerical simulations (Jacobs and Durbin, 2001; Lardeau et al., 2007; Nolan and Zaki, 2013), and theoretical analysis (Leib et al., 1999) (hereafter referred to LWG99) (Goldstein, 2014; Luchini, 2000; Ricco et al., 2011; Wundrow and Goldstein, 2001). The boundary layer has been revealed to act as a filter for the full-spectrum free-stream vortical disturbances, thereby allowing low frequency disturbances to penetrate into the boundary-layer core and to

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amplify significantly, whilst preventing high-frequency fluctuations from growing downstream. The high-frequency disturbances therefore remain confined in the outer portion of the shear layer. As the breakdown mechanism is still unclear, prediction of transition subjected to free-stream turbulence for aeronautical and turbomachinery applications relies heavily on empirical models based mainly on experimental data (Abu-Ghannam and Shaw, 1980; Mayle, 1991; Menter et al., 2006).

1.2. Theoretical frameworks for the laminar streaks

A mathematical description of the Klebanoff modes has been developed by Goldstein and co-workers (LWG99) (Goldstein, 2014; Wu and Choudhari, 2003; Wu and Luo, 2003; Wundrow and Goldstein, 2001). The Goldstein theory accounts for the presence of free-stream disturbances that are responsible for the generation and downstream development of the streaks. It is remarked that the correct description of the free-stream vortical disturbances is essential for capturing the streak dynamics. The key physical mechanism for the formation of the Klebanoff modes is revealed: there is an interaction between the boundary-layer disturbances and the free-stream fluctuations and the free-stream flow is continuously affected by the downstream growth of the boundary layer. Only the wall-normal and spanwise velocity components of the free-stream disturbances are relevant for the formation and growth of large streamwise-velocity streaks in the boundary-layer core, while the free-stream streamwise velocity component plays a secondary role. Ricco (2009b) (hereafter referred to as R9), by adopting LWG99's asymptotic approach, showed that a balance between the free-stream pressure fluctuations and the streamwise velocity fluctuations is relevant for the formation of the streak fluctuations in the outer part of the boundary layer. A realistic streak profile along the whole wall-normal extent of the boundary layer was obtained by R9 and good agreement with the experimental data by Westin et al. (1994) was shown. Ricco and Dilib (2010) employed R9's approach to show that the peak of the streak amplitude in the boundary-layer core may be completely suppressed if intense wall transpiration is applied and Ricco and co-authors (Ricco et al., 2013, 2009; Ricco and Wu, 2007) have studied the Klebanoff modes in compressible boundary layers.

Other theories have been put forward to describe the laminar streaks. The three most relevant ones are discussed below.

- *The Taylor–Stewartson theory*
  The first attempt to model the wall-normal profile of the streaks is due to Taylor (1939). He recognized that the streak profile can be described in terms of a small spanwise modulation of the boundary layer thickness. Stewartson (1957) translated Taylor's original idea in mathematical form by a simple perturbation of the boundary-layer thickness and showed that the root-mean-square (rms) profile of the streamwise velocity fluctuations agrees well with $u = \eta F''$ (where $F(\eta) = \psi/\sqrt{2\nu U_\infty x}$ is the Blasius function (Batchelor, 1967), ψ is the boundary-layer streamfunction, ν is the kinematic viscosity, U_∞ is the free-stream velocity, x is the streamwise coordinate, $\eta = y/\delta$ is the scaled wall-normal coordinate and δ is the laminar boundary-layer thickness).
- *The optimal-growth theory*
  This approach was first developed by Luchini (2000) and Andersson et al. (1999). The objective of the analysis is to find the initial velocity profile near the leading edge that maximizes a specified cost function, which may represent the energy of the perturbation within the entire viscous layer or at a specified downstream distance. When the peak disturbance is normalized, the wall-normal profile agrees well with the Taylor–Stewartson mode and with the experimental rms profiles of the streamwise velocity in the boundary-layer core (Matsubara and Alfredsson, 2001; Westin et al., 1994), although no comparison on the downstream evolution has been carried out.

  The three-dimensional boundary-layer equations (which coincide with LWG99's boundary-region equations) describe the flow. The crucial difference with the Goldstein theory is the specification of the *initial* and *free-stream boundary* conditions. In the optimal-growth theory, the free-stream disturbances, which cause the formation and downstream growth of the streaks, are not included in the formulation. Homogeneous outer boundary conditions are imposed on the streamwise and spanwise velocity components, i.e., the boundary-layer disturbances vanish as the boundary-layer wall-normal coordinate approaches the free stream. As a consequence, the initial conditions in the general three-dimensional case cannot be found before starting the numerical integration. A special case is represented by disturbances with spanwise wavelengths which are much larger than the boundary-layer thickness. For this case the downstream growth rate is $x^{0.213}$ (Luchini, 1996).

  Goldstein's asymptotic analysis instead allows the mathematically precise and unambiguous specification of both the *initial* conditions, which describe the flow in the proximity of the leading edge of the plate from which the downstream evolution of the streaks commences, and the *outer (free-stream)* boundary conditions, which characterize the flow in the outer portion of the boundary layer and its interplay with the free-stream disturbance flow. The correct mathematical representation of both conditions is crucial because they uniquely determine the streak dynamics. The initial conditions are unequivocally linked with the outer flow through the matched asymptotic expansion approach. The key point here is that their mathematical relation synthesizes the physical interaction between the oncoming free-stream disturbance flow and the boundary layer near the leading edge. The initial and outer flow are therefore fully consistent and their mathematical relationship is found by expanding the solution of the outer flow through a series (LWG99). This series in turn leads to a regular power-series expansion of the initial flow near the leading edge. An asymptotic composite solution for the wall-normal streak profile at small downstream distances is then obtained. This profile is used to initiate the downstream computation of the parabolic boundary-region equations.

  In the optimal-growth theory, the mathematical relationship between the boundary-layer perturbation flow and the free-stream flow is not established. The initial conditions *are therefore unknown a priori* and thus cannot be specified as an *input* to the calculations. The initial condition is computed as an *output* through the optimization procedure, together with the streak flow field downstream. As elucidated by Wundrow and Goldstein (2001), the effectiveness of the optimal growth theory to model the early stage of bypass transition is thus questioned.

  As the oncoming free-stream perturbation flow is neglected in the framework, the "optimal" flow is obviously not dependent on the free-stream flow characteristics (such as wavelength, frequency, energy spectrum) as it is in experiments and in the Goldstein theory. Instead, the final solution (initial and downstream flows) depends on the arbitrary choices of the cost function to be maximized (velocity components to be included, kernel), on the initial location, and on the final location of the domain inside which the cost function is maximized. Through the optimal-growth theory, the characteristics of the free-stream flow, such as its wavelength, frequency, intensity, and spectrum, cannot be linked with the downstream growth of the streaks, with the location of their secondary instability and with the dynamics of bypass transition.