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Oscillating characteristic of free surface from stability to instability of thermocapillary convection for high Prandtl number fluids

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ABSTRACT

An investigation on oscillating characteristic of free surface in a liquid bridge for high *Pr* number fluids under gravity has been conducted numerically. Against the former studies, free surface is treated as a deformable surface in the direct numerical simulation (DNS) of liquid bridge by using a mass conserving level set method for the first time. The results show that, the moving track of vortex centers appears the shape of axisymmetric hook in the stable stage of thermocapillary convection, and the fluctuation of surface velocity is closely relative to the motion of cell flow toward the center and cold disk. In the process of transforming from the stable stage to oscillatory stage, the moving track of vortex centers of cell flow shows the asymmetry and step characteristic. The coupling effects of the temperature, velocity and free surface oscillations constitute complete mechanism of thermocapillary convection oscillations. The temperature firstly oscillates at the hot corner, and transfer direction of temperature oscillation is from the hot corner to inner. The velocity of surface flow is larger than that of internal flow, and the surface flow is supplied by the bulk return flow with the disturbance information of the velocity. The velocity oscillation lags behind the temperature oscillation at the hot corner. The closer to the intermediate height, the larger amplitudes of temperature and velocity are. The oscillation law of velocity and free surface at the intermediate height are the same, and there are two obvious oscillation bifurcations.

1. Introduction

The convective motion along a free surface caused by changed surface tension is called as Marangoni convection. This change can be caused by many reasons, and the parts of them caused by temperature are called as thermocapillary convection. During the floating zone process for single-crystal, thermocapillary convection plays an important role on the quality of crystal material (Croll et al., 1986; Eyer et al., 1985; Schwabe et al., 1981). The crystal growth by zone melting method in microgravity environment is considered to be an important method to prepare high quality crystal, thus the study on thermocapillary convection in the floating zone has become one of important topics. Chang and Wilcox (1975) proposed a hydrodynamic model of liquid bridges for thermocapillary convection in the floating zone. This model was used for the study on convective characteristics and mechanism inside melting zone, not involving phase change process on solid–liquid interface. The experiment of thermocapillary convection for liquid bridges of half floating zone was conducted by Chun (1980), and Schwabe et al. (1978), and the oscillatory thermocapillary convection was found in melting zone. Such oscillatory convection brings corresponding oscillatory temperature field which relates to the growth of striation in the crystal and affects the growth quality of crystal. Since 1980, the researches about thermocapillary convection have focused on the critical parameters of oscillation, vibration mechanism and measures to control the oscillation.

At present, there have been a lot of numerical simulations and theoretical analyses on flow and oscillation mechanism in liquid bridges for low *Pr* number fluid (Hibiya et al., 2008; Levenstam et al., 1995; Bazzi et al., 2000; Davis et al., 2008; Kuhlmann et al., 1993). While in high *Pr* number range, the experimental and numerical investigations for oscillatory thermocapillary convection are still not enough (Montanero et al., 2008). Melnikov et al. (2014) studied formation of particle accumulation structures (PAS) in a supercritical flow driven by the combined effects of buoyancy and thermocapillary forces under earth's gravity. The accumulation of particles in coherent structures is possible only in a periodic oscillatory flow. Sato et al. (2013) found that an oscillatory flow emerges in the high aspect liquid bridge of 20 cSt silicone oil (Pr = 206.8) with the azimuthal mode number m = 1. As increasing the intensity of the thermocapillary effect in terms of the Marangoni number Ma, nonlinearity over the free surface is raised through unique propagations of the hydrothermal wave (HTW). The effect of interfacial heat transfer with ambient

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gas on the onset of oscillatory convection in a liquid bridge of large Prandtl number on the ground was systematically investigated by the method of linear stability analyses (Xun et al., 2009). Li et al. (2006) conducted an unsteady three-dimensional numerical simulation of thermocapillary convection in an encapsulated liquid bridge. The experiment was conducted on the sounding rocket MAXUS 4 launched from ESRANGE in Kiruna, North-Sweden. Schwabe observed the onset of hydrothermal waves for a 2 cSt silicone oil (Pr=28) liquid bridge and to measure their features such as the wave phase speed and the angle between the wave vector and the applied temperature gradient was reached (Schwabe, 2005).

It is difficult to find out accurate and stable analyses or numerical simulations on flow structures of high Pr number fluids. Meanwhile, there are still major disputes on data veracity (Han et al., 1996; Rupp et al., 1989). The reason is that the majority studies adopted the simplified model without taking the environment influence and the dynamic surface deformation into account. At the same time, existing methods for capturing micro surface migration of the liquid bridge cannot meet the resolution requirements of surface deformation under gravity (Wanschura et al., 1995). In this paper, the DNS of thermocapillary convection in a liquid bridge for high Pr number fluid under gravity has been conducted to explore the flow pattern in the liquid bridge. The governing equations of thermocapillary convection under gravity are given by the non-dimensional mass, Navier–Stokes and energy conservation equations and solved on a staggered grid.

2. Physical model and geometric model

The liquid bridge with radius R and height H is suspended between two coaxial disks and surrounded by the air in a rectangular container with height H and width $4R$ as shown in Fig. 3. The temperature difference between the two disks is $\Delta T = T_t - T_b$, where T_t and T_b are the temperature of the upper and bottom disks, respectively. The general governing equations of the problem under gravity are given by the following non-dimensional mass, Navier–Stokes and energy conservation equations.

$$\mathbf{u}_t + (\mathbf{u}\cdot\nabla)\mathbf{u} = \mathbf{g}_u + (-\nabla p + \nabla\cdot(2\mu\mathbf{D})/Re + (1-\mathrm{Ca}\theta)\kappa\delta(d)\mathbf{n}/We)/\rho. \quad (1)$$

$$\nabla\cdot\mathbf{u} = 0. \quad (2)$$

$$\partial\theta/\partial t + \nabla\cdot(\mathbf{u}\theta) = \nabla^2\theta/Ma. \quad (3)$$

where $\mathbf{u} = (u, v)$ is the fluid velocity, u ($u = u_s/U$) and v ($v = v_s/U$) are dimensionless transverse and vertical velocity, respectively, U ($|\sigma'_T|\Delta T/\mu$) is characteristic velocity, where ΔT is temperature difference ($\Delta T = T_t - T_b$), θ is dimensionless temperature and we take $\theta = (T - T_b)/(T_t - T_b)$, $\rho = \rho$ ($\mathbf{x}$,t) is the fluid density, $\mu = \mu$ ($\mathbf{x}$,t) is the fluid viscosity, $\mathbf{D}$ is the viscous stress tensor, κ is the curvature of the interface, d is the normal distance to the interface, δ is the Dirac delta function, $\mathbf{n}$ is the unit normal vector at the interface, t is the dimensionless time, we denote $t = t_s U/\bar{L}$, where t_s is dimensional time, $\bar{L}$ is the characteristic length and we take $\bar{L} = 2R$. In addition, x and y are dimensionless coordinates. We denote $x = X/\bar{L}$ and $y = Y/\bar{L}$. The surface tension coefficient is considered to be a linearly function of temperature and defined as $\sigma = \sigma_c - \sigma_T(T - T_b)$, where σ_c is a reference value of surface tension and σ_T is the temperature coefficient of surface tension. We denote $\sigma'_T = \partial\sigma/\partial T$, and T is the temperature.

The key parameters are ρ_g/ρ_l and μ_g/μ_l, the dimensionless density and viscosity ratio, respectively. ρ_l and μ_l are the dimensional density and viscosity of the liquid bridge, respectively, while ρ_g and μ_g are the dimensional density and viscosity of the ambient air, respectively. $Re = \rho_l U_\infty \bar{L}/\mu_l$, Reynolds number, $We = \rho_l U_\infty^2 \bar{L}/\sigma$, Weber number, $Pr = \mu_l/\rho_l\alpha$, Prandtl number, $Ca = \mu_l U_\infty/\sigma$, Capillary number, $Ma = \sigma_T\Delta T\bar{L}/\mu_l a = RePr$, Marangoni number, $\theta = (T - T_b)/\Delta T$, excess temperature, α is the thermal diffusivity, and $\mathbf{g}_u$ represents a unit gravitational force. The relative importance of buoyancy and thermocapillary effects is determined by Bond number, $B = \rho g\beta\bar{L}^2/\sigma_T$, β is the coefficient of thermal expansion.

The outer boundary for air region would be satisfied:

$$\theta = 0 \quad (y = 0), \quad (4)$$

$$\theta = 1.0 \quad (y = 0.5). \quad (5)$$

In the two phase system studied here, the initially stationary liquid bridge was considered with the initial velocity for both liquid bridge and ambient air,

$$\mathbf{u}(t = 0) = 0. \quad (6)$$

The non-slip condition was used for all walls of the computational domain,

$$\mathbf{u} = 0. \quad (7)$$

The level set method was originally introduced by Osher and Sethian (1988) to numerically predict the moving interface Γ(t) between two fluids. Instead of explicitly tracking the interface, the level set method implicitly captures the interface by introducing a smooth signed distance from the interface in the entire computational domain. The level set function $\phi(\mathbf{x}$,t) is taken to be positive outside the liquid bridge, zero on the interface and negative inside the liquid bridge. The interface motion is predicted by solving the following convection equation for the level set function of $\phi(\mathbf{x}$,t) given by:

$$\phi_t + \mathbf{u}\cdot\nabla\phi = 0, \quad (8)$$

$$\mathbf{u}\cdot\nabla\phi = (u\phi)_x + (v\phi)_y, \quad (9)$$

$$\begin{aligned}(u\phi)_x + (v\phi)_y = {} & (u_{i+1/2,j} + u_{i-1/2,j})(\phi_{i+1/2,j} - \phi_{i-1/2,j})/(2h) \\ & + (\phi_{i+1/2,j} + \phi_{i-1/2,j})(u_{i+1/2,j} - u_{i-1/2,j})/(2h) \\ & + (v_{i,j+1/2} + v_{i,j-1/2})(\phi_{i,j+1/2} - \phi_{i,j-1/2})/(2h) \\ & + (\phi_{i,j+1/2} + \phi_{i,j-1/2})(v_{i,j+1/2} - v_{i,j-1/2})/(2h).\end{aligned} \quad (10)$$

For smooth data, we have$(\phi_{i+1/2,j} + \phi_{i-1/2,j}) \approx (\phi_{i,j+1/2} + \phi_{i,j-1/2})$. In addition, we have $(u_{i+1/2,j} - u_{i-1/2,j}) \approx (v_{i,j+1/2} - v_{i,j-1/2})$ because $\mathbf{u}$ is numerically divergence free.

Thus,

$$\begin{aligned}(u\phi)_x + (v\phi)_y \approx {} & (u_{i+1/2,j} + u_{i-1/2,j})(\phi_{i+1/2,j} - \phi_{i-1/2,j})/(2h) \\ & + (v_{i,j+1/2} + v_{i,j-1/2})(\phi_{i,j+1/2} - \phi_{i,j-1/2})/(2h),\end{aligned} \quad (11)$$

For computing $u_{i+1/2,j}$(similarly for $u_{i,j+1/2}$, $\phi_{i+1/2,j}$,...), a second-order ENO scheme is used as follows:

Define

$$m(a,b) \equiv \begin{cases} a & \text{if } |a| \le |b| \\ b & \text{otherwise} \end{cases}. \quad (12)$$

Let

$$u_L \equiv u_{i,j} + m(u_{i+1,j} - u_{i,j}, u_{i,j} - u_{i-1,j})/2, \quad (13)$$

$$u_R \equiv u_{i+1,j} - m(u_{i+2,j} - u_{i+1,j}, u_{i+1,j} - u_{i,j})/2, \quad (14)$$

$$u_M \equiv (u_L + u_R)/2. \quad (15)$$