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Triple diffusive mixed convection along a vertically moving surface

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ABSTRACT

This paper presents a numerical investigation on steady triple diffusive mixed convection boundary layer flow past a vertical plate moving parallel to the free stream in the upward direction. The temperature of the plate is assumed to be hotter compared to the surrounded fluid temperature. Sodium chloride and Sucrose are chosen as solutal components which are added in the flow stream from below with various concentration levels. The concentrations of NaCl-Water and Sucrose-Water are considered to be higher near the wall compared to the concentrations of NaCl-Water and Sucrose-Water within the free stream. The coupled nonlinear partial differential equations are transformed using the non-similarity variables and solved numerically by an implicit finite difference scheme with quasi-linearization technique. The effects of Richardson numbers, velocity ratio parameters, ratio of buoyancy parameters and Schmidt numbers of both the solutal components on the fluid flow, thermal and species concentration fields are investigated. Results indicate that the species concentration boundary layer thickness decreases with the increase of Schmidt numbers and that increases with the ratio of buoyancy parameters for both the species components. Overall, the mass transfer rate is found to increase with Schmidt numbers approximately 4.36% and 64.56% for NaCl and Sucrose, respectively.

1. Introduction

Combined heat and mass transfer also known as thermosolutal convective problem has experienced vigorous growth in recent years [1]. The convective heat and mass transfer in the boundary layer involving a continuously moving plate in a parallel free stream has several industrial and engineering applications such as oil recovery [2], drying process [3], cooling of metallic plates [4], distillation process [5], solidification [6], the aerodynamic extrusion of plastic sheets [7], condensation processes [8] to name a few.

A significant research has been performed to study the fluid and heat flow during double diffusive mixed convection within the boundary layer regime. Bansod [9] analyzed the effects of suction and blowing during double diffusive mixed convection over an inclined permeable surface in porous media. Double diffusive convection near a vertical truncated cone in fluid saturated porous media with viscosity proportional to an inverse linear function of temperature was investigated by Cheng [10]. Subhashini et al. [11] carried out numerical study on double diffusive mixed convection boundary layer flow over a permeable surface involving convective surface boundary condition using the local similarity method. Patil et al. [12] dealt with non-similar solutions for the steady double diffusive mixed convection boundary layer flow along a semi-infinite vertical surface. Further, Patil et al. [13] extended their study for double diffusive mixed convection boundary layer flow along an impermeable exponentially stretching surface in the presence of Soret and Dufour effects and chemical reactive species. Double diffusive mixed convection boundary layer flow of a magneto-micropolar fluid over a wedge involving convective boundary condition in presence of chemical reaction was investigated by Swapna et al. [14]. Beg et al. [15] analyzed the unsteady magnetohydrodynamic double diffusive mixed convection boundary layer flow along a vertical porous sheet in presence of chemical reaction. A study on steady mixed convection flow from a vertical stretching sheet with variable temperature and concentration was carried out by Patil [16]. Further, Patil et al. [17] have analyzed the double diffusive mixed convection flow along an exponentially stretching surface in presence of viscous dissipation.

Recently, research has been focused on triple diffusive boundary layer flow where the change in density occurs due to thermal

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Nomenclature

C	species concentration
C_f	local skin-friction coefficient
C_p	specific heat capacity
D	mass diffusivity
f	dimensionless stream function
F	dimensionless velocity
g	acceleration due to gravity, m s^{-2}
G	dimensionless concentration for NaCl
Gr	Grashof numbers
H	dimensionless concentration for Sucrose
k	thermal conductivity, W m^{-1} K^{-1}
L	characteristic length, m
Nu	local Nusselt number
Pr	Prandtl number
Re	Reynolds number
Ri	Richardson number
Sc	Schmidt number
Sh	Sherwood number
T	temperature of the fluid, K
u	x component of velocity, m s^{-1}
U	composite reference velocity, m s^{-1}
U_w	moving plate wall velocity, m s^{-1}
U_∞	free stream velocity, m s^{-1}
v	y component of velocity, m s^{-1}
x	distance along x coordinate, m
y	distance along y coordinate, m

Greek symbols

α	thermal diffusivity, m^2 s^{-1}
β_T	volume expansion coefficients, K^{-1}
β_C	solutal expansion coefficient, mol litre^{-1}
ε	ratio of free stream velocity to the reference velocity
μ	dynamic viscosity, kg m^{-1} s^{-1}
ν	kinematic viscosity, m^2 s^{-1}
ρ	density, kg m^{-3}
ψ	dimensionless stream function
θ	dimensionless temperature
ξ, η	transformed variables

Subscripts

s_1, s_2	first and second species concentration, respectively
w, ∞	conditions at the wall and infinity, respectively
ξ, η	partial derivatives with respect to these variables, respectively

diffusivity as well as two different molecular diffusivities. Triple diffusive convection has relevant applications in the underground water flow, contaminant transport and warming of the stratosphere. A pioneering analytical work on the linear stability analysis on triple diffusive convection in a horizontal fluid layer with free boundary conditions was performed by Griffiths [18]. Also, Griffiths [19] experimentally obtained the criteria for the formation of salt fingers and diffusive interface during triple diffusive convection in a horizontal fluid layer. Further, the measurements of flux for a three component thermohaline interface was carried out by Griffiths [20]. The experiment was carried out for the fluid solution of three salts KCl, $NaCl$ and $MgCl_2$ and the fluid layer was heated from below and cooled from above. Theoretical investigations on stability analysis involving triple diffusive convection flow in a porous horizontal layer was carried out by Poulikakos [21]. It is observed from the study that the porosity of the medium has significant effect on the stability of boundaries. Pearlstein et al. [22] examined the onset of convective instability in a triple diffusive fluid layer subject to the boundary conditions similar to [18]. Their study reveals that, the multi-valued nature of the stability boundaries become complex due to the presence of the heart-shaped oscillatory neutral curves. In addition, Terrones and Pearlstein [23] extended the convective stability analysis for diffusive flows in multi-components fluid layer. The types and behavior of multiple instabilities during two-dimensional triple diffusive convection flows was studied by Moroz [24]. Lopez et al. [25] analyzed the linear stability criteria in triple diffusive fluid layer enclosed between the rigid boundaries. Various aspects of crossed diffusion on the linear stability analysis in a fluid medium involving three different components was studied by Terrones [26]. Straughan and Walker [27] studied the stability criteria involving penetrative convection in a triply diffusive system using Chebyshev-tau method. In another study, Straughan and Tracey [28] examined the effect of various boundary conditions on the instability problem for the triple-diffusive convection in a fluid layer in presence of a heat source.

Rionero [29] carried out stability analysis of the triple diffusive convection flow in a horizontal fluid layer heated from below and salted from both above and below by two distinct salts. Further, Rionero [30] generalized the theory of stability criteria for multicomponent fluid mixture saturated in porous media. The triple diffusive boundary layer natural convection flow past a horizontal flat plate embedded by porous saturated nanofluid was studied by Khan et al. [31]. Further, Khan et al. [32] examined triple diffusive boundary layer flow past a horizontal plate embedded by porous saturated fluid medium using Darcy model with similar boundary conditions of [31]. The effect of couple stresses on linear and weakly nonlinear stability of a fluid layer in presence of the two different salts was investigated by Shivakumara and Naveen Kumar [33]. Prakash et al. [34] investigated the linear stability of a triply diffusive fluid layer rotating vertically with a uniform velocity.

All of the aforementioned studies reveal that, the role of moving vertical surface on heat and mass transfer rate during triple diffusive convective flow has not yet been addressed in literature. Thus, the aim of the current study is to analyze the mixed convection boundary layer flow past a vertical moving surface embedded by fluid mixture composed of two different chemical components. Triple diffusive boundary layer flow has practical applications in food processing, underground water flow, contaminant transport, worming of the stratosphere, acid rain effects [27,28,35–37]. The focus of this study is to obtain the non-similar solutions. The detailed analysis has been carried out using finite difference method.

2. Governing equations, boundary conditions and numerical simulations

The physical domain of a steady incompressible mixed convection boundary layer flow along a semi-infinite moving vertical plate is shown in Fig. 1. In the two dimensional flow field, the x-axis is considered along the moving plate in the vertically upward direction whereas the y-axis is taken normal to it. It is assumed that the plate velocity (U_w) and free stream velocity (U_∞) are constants and act in the upward direction [38]. The temperature of the moving wall (T_w) is kept as constant. The concentration of Sodium Chloride (NaCl) and Sucrose near the wall is assumed to be C_{1w} and C_{2w}, respectively. All the thermophysical properties of the fluid in the flow model are assumed to be constant except the density of the fluid. The Boussinesq approximation is invoked to relate density changes to temperature and