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A direct solution for radiative intensity with high directional resolution in isotropically scattering media

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Radiative intensity with high directional resolution can provide fruitful information inside radiative systems, which is very useful for inverse analysis. In this work, a direct solution for radiative intensity with high directional resolution in isotropically scattering media enclosed by diffuse boundaries is developed. First, linear equations about radiative intensity at the boundaries (outgoing directions) and source terms in the media are established based on the integral form of the Radiative Transfer equation (RTE). Then, after directly solving the established equations and with the obtained radiative intensity and source terms, radiative intensity at any position with high directional resolution is readily calculated by summation operation. The proposed method is validated by comparing the calculated directional radiative intensity with that of the iterative Distributions of Ratios of Energy Scattered Or Reflected (iterative-DRESOR) method as well as the reverse Monte Carlo (RMC) method, both in one-dimensional and three-dimensional cases. The computing time comparison shows that the proposed method has a distinct advantage for calculating radiative intensity with high directional resolution compared with the reverse Monte Carlo method.

1. Introduction

The behavior of radiative heat transfer in participating media is governed by the Radiative Transfer Equation (RTE), which describes radiative intensity change along a specific direction in the media. Up to now, there are many widely used solutions for RTE, such as the Monte Carlo method [1,2], the discrete ordinates method (DOM) [3,4], the spherical harmonics method [5–7], and the zonal method [8,9]. Meanwhile, in last twenty years or so, while radiative heat transfer is widely considered in different applications, many other solutions are also developed, such as the finite volume method [10,11], the finite element method [12–15], the spectral method [16–19], the lattice Boltzmann method [20–23], to name but a few. Most of the work mentioned above focus on the integral radiative quantities, and the solutions developed in those work can only obtain integral radiative quantities or radiative intensity in very limited directions. It is very difficult or inefficient to calculate directional radiative intensity in many directions.

In recent years, temperature and/or species concentration detection in combustion systems using radiative images became popular [24–30]. This technology provides useful information of combustion status in furnaces and very helpful for combustion adjustment and making good use of fuel. The radiative images are normally captured by charge-coupled device (CCD) cameras, which are very small receivers with many pixels corresponding to high resolution directions. Then, calculating radiative intensity with high directional resolution onto a small receiver is essential to analyze the flame images captured by the CCD cameras. Li et al. [31,32] developed the discrete ordinate scheme with (an) infinitely small weight(s) (DOS + ISW), and this method can calculate radiative intensity in arbitrary directions. In a similar way, Wei et al. [33] proposed a modified spectral method for simulating arbitrary directional radiative intensity in participating media with graded refractive index. In these methods, new directions in which the radiative intensity is required are added to an existing discrete ordinate quadrature set, and the weights associated with these new discrete directions are set to be infinitely small. Since the calculation of radiative intensity in the new directions are all based on the original discrete ordinate quadrature set, the applicability and accuracy of these methods are constrained by those of the traditional discrete ordinates method or spectral method. For example, both methods suffer from ray effects due to the limited discretized directions in the original directional quadrature set.

Reverse Monte Carlo (RMC) method is capable of solving complex problems and have good accuracy if sufficient energy bundles are used. The energy bundles are traced from the detector to the

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radiative source, which makes the tracing process more efficient than the traditional Monte Carlo method [34]. It has been applied for radiative image analysis and reconstructing the three-dimensional temperature distribution in combustion systems [28,29]. Distributions of Ratios of Energy Scattered Or Reflected (DRESOR) method is based on the Monte Carlo method and is another method that has been applied for radiative image analysis to obtain three-dimensional temperature [25,26]. In the tracing process of energy bundles, the directional distributions of energy scattered by the media or reflected by the boundaries are recorded and used for directional radiative intensity calculation [35–38]. It shows higher efficiency than the RMC method if radiative intensity in hundreds of directions or more are needed [38], which is usually the case of the temperature detection technology by radiative image analysis. In order to mitigate the drawbacks of the Monte Carlo method, which are large computation time and unavoidable statistical errors, Wang et al. [39–41] proposed to use equation solving method to calculate DRESOR values instead of using the Monte Carlo sampling in the DRESOR method. The method is named equation solving DRESOR (ES-DRESOR) method, and this method shows higher efficiency for the radiative image analysis comparing with the RMC and DRESOR methods [41]. However, the introducing of the DRESOR values makes the theory of this kind of methods complicated. Also, if the real-time radiative transfer needs to be solved to improve the accuracy of the temperature detection technology, a more efficient method to calculate radiative intensity with high directional resolution is still needed.

In this work, a direct solution for radiative intensity with high directional resolution in isotropically scattering media enclosed by diffuse boundaries is developed. Linear equations about radiative intensity at the boundaries (outgoing directions) and source terms in the media are established. After solving the linear equations directly, radiative intensity at the boundaries (outgoing directions) and source terms in the media are obtained. With those calculated quantities radiative intensity at any position in any direction can be calculated. The theoretical development of this method is given in Section 2. The accuracy and computing efficiency of this method is tested in Section 3. Finally, some conclusions are given.

2. Theoretical development

2.1. Establishment of linear equations based on the RTE

The integral form of the radiative transfer equation for an emitting, absorbing and scattering medium is [34]

$$I(\mathbf{r},\hat{\mathbf{s}}) = I_w(\mathbf{r}_w,\hat{\mathbf{s}})\exp\left[-\int_0^s \beta ds''\right] + \int_0^s S(\mathbf{r}',\hat{\mathbf{s}})\exp\left[-\int_0^{s'} \beta ds''\right]\beta ds' \tag{1}$$

The medium is assumed to be gray in this work. However, the relations also hold on a spectral basis for nongray media. While isotropically scattering media enclosed by diffuse boundaries are considered in this work, Eq. (1) turns to

$$I(\mathbf{r},\hat{\mathbf{s}}) = I_w(\mathbf{r}_w)\exp\left[-\int_0^s \beta ds''\right] + \int_0^s S(\mathbf{r}')\exp\left[-\int_0^{s'} \beta ds''\right]\beta ds' \tag{2}$$

where β is the extinction coefficient of the medium, $I_{\rm w}(\mathbf{r}_{\rm w})$ is the outgoing radiative intensity at the boundary, $S(\mathbf{r}')$ is the radiative source term, they are given as [34]

$$I_w(\mathbf{r}_w) = \varepsilon(\mathbf{r}_w)I_b(\mathbf{r}_w) + \frac{\rho(\mathbf{r}_w)}{\pi}\int_{\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}'<0} I(\mathbf{r}_w,\hat{\mathbf{s}}')|\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}'|d\Omega' \tag{3}$$

$$S(\mathbf{r}') = (1-\omega)I_b(\mathbf{r}') + \frac{\omega}{4\pi}\int_{4\pi} I(\mathbf{r}',\hat{\mathbf{s}}')d\Omega' \tag{4}$$

where $\hat{\mathbf{n}}$ is the normal direction of the boundary, $\rho(\mathbf{r}_w)$ is the diffuse reflectivity of the boundary, ω is the scattering albedo of the medium, $I_b(\mathbf{r}_w)$ and $I_b(\mathbf{r}')$ are blackbody radiative intensity of the boundary and the medium, respectively.

$I(\mathbf{r}_w,\hat{\mathbf{s}}')$ at right hand side of Eq. (3) is incoming radiative intensity at the boundary, it varies with different incident directions. According to Eq. (2), it can be rewritten as

$$I(\mathbf{r}_w,\hat{\mathbf{s}}') = I_w(\mathbf{r}'_w)\exp\left[-\int_0^s \beta ds''\right] + \int_0^s S(\mathbf{r}')\exp\left[-\int_0^{s'} \beta ds''\right]\beta ds' \tag{5}$$

Substitute Eq. (5) into Eq. (3), we have

$$I_w(\mathbf{r}_w) = \varepsilon(\mathbf{r}_w)I_b(\mathbf{r}_w) + \frac{\rho(\mathbf{r}_w)}{\pi}\int_{\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}'<0}\left\{I_w(\mathbf{r}'_w)\exp\left[-\int_0^s \beta ds''\right] + \int_0^s S(\mathbf{r}')\exp\left[-\int_0^{s'}\beta ds''\right]\beta ds'\right\}_{\hat{\mathbf{s}}'}|\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}'|d\Omega' \tag{6}$$

In a similar way, $I(\mathbf{r}',\hat{\mathbf{s}}')$ at the right hand side of Eq. (4) can be updated by Eq. (2), thus Eq. (4) may be rewritten as

$$S(\mathbf{r}) = (1-\omega)I_b(\mathbf{r}) + \frac{\omega}{4\pi}\int_{4\pi}\left\{I_w(\mathbf{r}_w)\exp\left[-\int_0^s \beta ds''\right] + \int_0^s S(\mathbf{r}')\exp\left[-\int_0^{s'}\beta ds''\right]\beta ds'\right\}_{\hat{\mathbf{s}}'} d\Omega' \tag{7}$$

Notice that the position vector $\mathbf{r}'$ in Eq. (4) is changed into $\mathbf{r}$ in Eq. (7) for deduction convenience. In Eqs. (6) and (7), it is obvious that, for a radiative system with an emitting, absorbing, and isotropically scattering medium and diffuse boundaries, if the geometry of the system and the radiative property distributions of the boundaries (ε, ρ) and the medium (ω, β) are specified, outgoing radiative intensity at the boundary $I_{\rm w}(\mathbf{r}_{\rm w})$ and radiative source term in the medium $S(\mathbf{r})$ are the only unknown quantities. If the radiative system is divided into N_w wall elements and N_s space elements, Eqs. (6) and (7) turns into $N_w + N_s$ linear equations with $N_w + N_s$ unknown quantities, which are readily solved to obtain $I_{\rm w}(\mathbf{r}_{\rm w})$ and $S(\mathbf{r})$. Then, radiative intensity $I(\mathbf{r},\ \hat{\mathbf{s}})$ at any position $\mathbf{r}$ in any direction $\hat{\mathbf{s}}$ can be calculated through Eq. (2).

2.2. Discretization of the equations and coefficient calculation

The radiative system is divided into N elements, which includes N_w wall elements and N_s space elements. The wall elements are labeled as $i_w = 1, 2, \ldots, N_w$ and the space elements are labeled as $i_s = N_w+1, N_w+2, \ldots, N_w+N_s$. The whole 4π space is divided into M discrete directions and labeled as $j = 1, 2, \ldots, M$. The discretized form of Eqs. (6) and (7) is

$$I_w(i_c) = \varepsilon(i_c)I_b(i_c) + \frac{\rho(i_c)}{\pi}\sum_{i=i_w}\left(\sum_{\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}_j<0} e^{-\tau_{ij}}|\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}_j|\Delta\Omega_j\right)\cdot I_w(i) + \frac{\rho(i_c)}{\pi}\sum_{i=i_s}\left(\sum_{\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}_j<0}(e^{-\tau_{ij1}} - e^{-\tau_{ij2}})|\hat{\mathbf{n}}\cdot\hat{\mathbf{s}}_j|\Delta\Omega_j\right)\cdot S(i) \tag{8}$$

$$S(i_c) = [1-\omega(i_c)]\cdot I_b(i_c) + \frac{\omega(i_c)}{4\pi}\sum_{i=i_w}\left(\sum_{j=1}^{M} e^{-\tau_{ij}}\Delta\Omega_j\right)\cdot I_w(i) + \frac{\omega(i_c)}{4\pi}\sum_{i=i_s}\left(\sum_{j=1}^{M}(e^{-\tau_{ij1}} - e^{-\tau_{ij2}})\Delta\Omega_j\right)\cdot S(i) \tag{9}$$