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The extended Reynolds analogy for the Couette problem: Similarity parameters

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ABSTRACT

We consider the extended Reynolds analogy for the Couette problem. That is, we study the relation between the shear stress and the energy flux transferred to the boundary surface at different velocities and temperatures. We use the direct simulation Monte Carlo (DSMC) method. We show that for all considered plates' velocities and temperatures the extended Reynolds analogy for a monatomic gas at any fixed Knudsen number depends on the plates' velocities and temperatures only via the Eckert number (up to statistical fluctuations). Additionally, we generalize an extended Reynolds analogy. For a monatomic gas in the transitional flow regime we show that the generalized Reynolds analogy up to statistical fluctuations depends only on Knudsen number for all considered Eckert numbers. For a gas at any Knudsen number as well as for liquid we show that the sum of the Reynolds analogies for the upper and lower plates in the Couette problem is exactly one.

1. Introduction

The Reynolds analogy, i.e., the proportionality of the shear stress and heat transfer in the Navier-Stokes Couette problem at arbitrary plate temperatures is well known [1]. In the case of equal plate temperatures this result leads to the fact that the heat fluxes transferred to the plate are equal to each other and are equal to half the product of the shear stress and the relative velocity of the plates [2].

Currently, due to the development of nanotechnology there is the increasing interest in heat transfer in micro channels [3–6]. In this connection, it is of interest to extend the Reynolds analogy to the case of a rarefied gas flows. For the rarefied gas Rayleigh problem, there exists the extended Reynolds analogy, i.e., the relation between the momentum flux and energy flux transferred to the plate [7].

The purpose of this paper is to extend the Reynolds analogy for the rarefied gas Couette problem. We consider the Couette problem mainly for a monatomic gas at an arbitrary Knudsen number.

This paper is organized as follows: In Section 2, we formulate the boundary value problem for the molecular velocities distribution function. In Section 3, we address the obtained results and their analyses. The results of the study are summarized in Section 4.

2. Problem statement

We consider the steady flow of a gas between two parallel plates located at a distance L with different temperatures T_1 and T_2. We associate the Cartesian coordinate system (x, y, z) with the first (lower) plate, thus making it in this coordinate system fixed. Let the second (upper) plate move relative to the first one with the velocity $\boldsymbol{U_r}$ parallel to the axis x. The x-axis direction coincides with the $\boldsymbol{U_r}$ direction. The y-axis is directed toward the upper plate. The flow is one-dimensional.

To describe the gas flow between the plates it is necessary to solve the boundary problem for the Boltzmann equation

$$v_y \frac{df}{dy} = J(f,f) \tag{2.1}$$

with the boundary conditions for the velocity distribution function f on the lower and upper plates

$$f = n_{r1}(2\pi R_{gas}T_1)^{-3/2} \exp\left[-\frac{v_x^2+v_y^2+v_z^2}{2R_{gas}T_1}\right], \quad v_y > 0, \quad y = 0 \tag{2.2}$$

$$f = n_{r2}(2\pi R_{gas}T_2)^{-3/2} \exp\left[-\frac{(v_x-U_r)^2+v_y^2+v_z^2}{2R_{gas}T_2}\right], \quad v_y < 0, \quad y = L. \tag{2.3}$$

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Here, $J(f, f)$ is the molecule collision integral [8]; v_x, v_y and v_z are the Cartesian velocity components of the molecules, R_{gas} is the gas constant. The distribution function parameters corresponding to the reflected molecules n_{r1} and n_{r2} can be found from the balance between the incident and reflected molecules.

To solve problem (2.1)–(2.3) it is necessary to specify the law of collisional interaction of molecules. We use two models of molecules: "Maxwell spheres" [9] with the $\sigma = \sigma_0/g$ interaction cross section, where σ_0 is a constant and g is a relative velocity of the colliding molecules and the "hard spheres" molecular model. For first model we have the typical free path of the molecules $\lambda = 3.2(\sqrt{\pi}n_{av}\sigma_0)^{-1}$, here and below $\sigma_0 = const$, n_{av} is the average density of the molecules between the plates, $c = \sqrt{R_{gas}(T_1 + T_2)}$, and for the second model $\lambda = (\sqrt{2}n_{av}\sigma_{sp})^{-1}$, where σ_{sp} = const.

The shear stress p_{xy} and the energy flux, transferred to the low plate E_1 are calculated using the distribution function:

$$p_{xy} = -\int m v_x v_y f(\mathbf{V})|_{y=0} d\mathbf{V}.$$

$$E_1 = -\int v_y \frac{m(v_x^2 + v_y^2 + v_z^2)}{2} f(\mathbf{V})|_{y=0} d\mathbf{V}, \quad \text{where}$$

$$d\mathbf{V} = dv_x dv_y dv_z,$$

m is the mass of the molecule.

We investigate the extended Reynolds analogy

$$R = \frac{E_1}{p_{xy}U_r}$$

From the dimensional analysis, it follows that R depends on not more than three dimensionless variables

$$T = \frac{T_1}{T_2}, U = \frac{U_r}{c}, \mathrm{Kn} = \frac{\lambda}{L}$$

and on the law of the collisional interaction of molecules.

3. Results and analysis

In the limit of small Knudsen numbers ($\mathrm{Kn} \to 0$) the fluid flow satisfies the Navier-Stokes equations and the no-slip boundary conditions on the plates [8]. The extended Reynolds analogy coincides in this limit with the Reynolds analogy obtained in [1]. After simple transformations this relationship takes the form

$$R = \frac{1}{2} - \frac{1}{\mathrm{PrEc}} \tag{3.1}$$

Here $Pr = \mu \cdot c_p/k$ is the Prandtl number; $\mathrm{Ec} = U_r^2/[c_p(T_1 - T_2)]$ is the Eckert number, μ is the dynamic viscosity, k is the thermal conductivity and c_p is specific heat at constant pressure. For a monatomic gas $Pr = 2/3$.

On the other hand, in the free molecular limit ($Kn \to \infty$), using the appropriate expressions for the shear stress and energy flux, it is easy to get that

$$R = \frac{1}{2} - \frac{2(\kappa - 1)}{\kappa}\frac{1}{\mathrm{Ec}} \tag{3.2}$$

Here κ is ratio of specific heats [$\kappa = \kappa(Pr)$]. For a monatomic gas $\kappa = 5/3$.

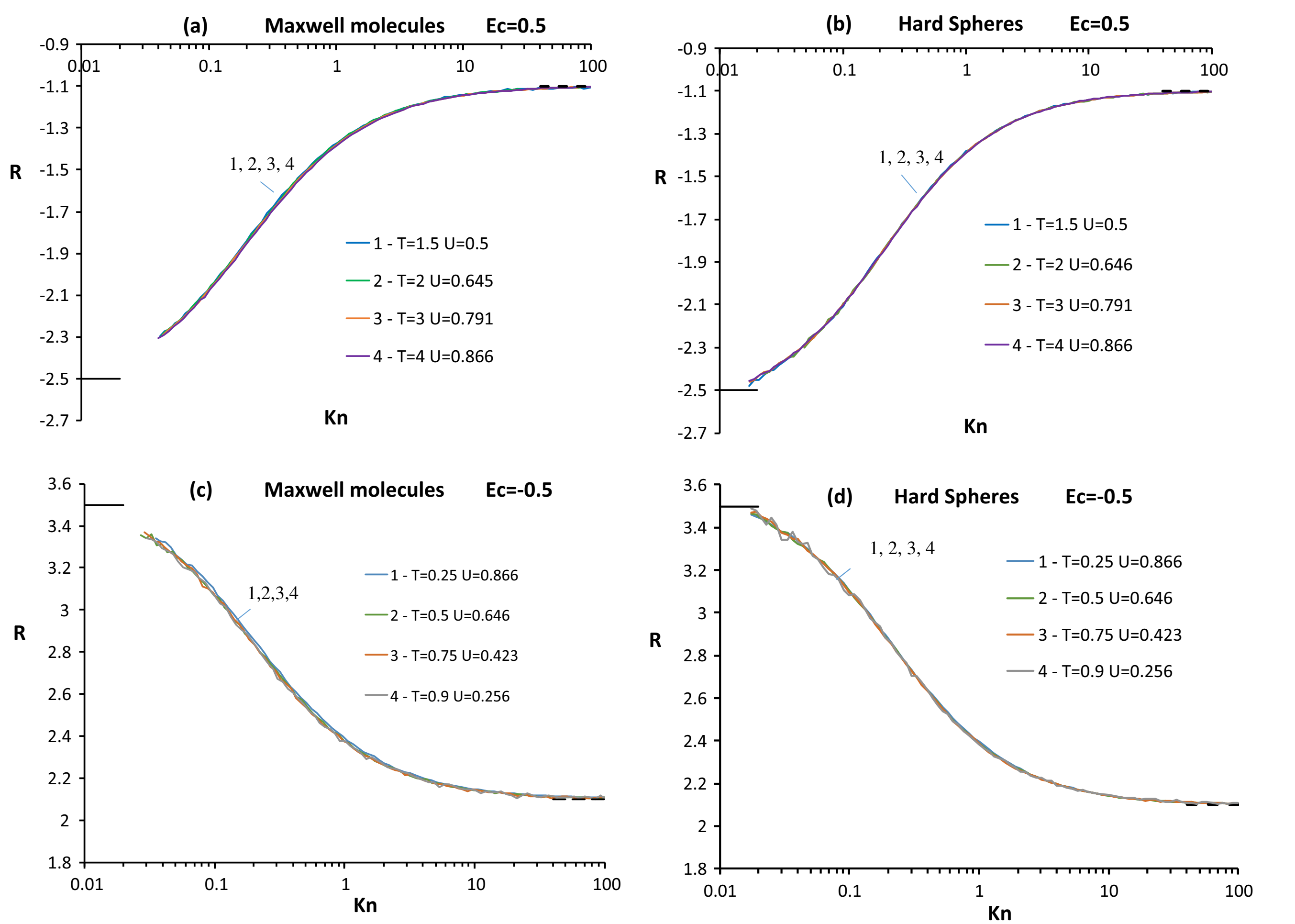

Fig. 1. The extended Reynolds analogy vs. Kn at different temperatures and velocities for $|\mathrm{Ec}| = 0.5$.