International Journal of Heat and Mass Transfer 117 (2018) 183–187

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Instabilities of thermocapillary flows between counter-rotating disks under microgravity conditions

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ARTICLE INFO

Article history:
Received 31 March 2017
Received in revised form 9 August 2017
Accepted 30 September 2017

ABSTRACT

Instabilities of thermocapillary flows between counter-rotating disks under microgravity conditions are investigated by linear stability analysis. The basic-state and perturbation equations are solved using the Chebyshev-collocation method. For small Prandtl number liquids ($Pr \leq 0.01$), bifurcation of thermocapillary flows between counter-rotating disks is found to be a 3D oscillatory state for the Coriolis number $\tau \leq 100$, except at certain Coriolis number where the most unstable perturbation is 3D stationary state. The critical capillary Reynolds number is a function of Prandtl number, Coriolis number and aspect ratio.

Energy analysis shows that the perturbation energy consists of the viscous dissipation, the work done by surface tension and the interaction between the perturbation flow and the basic flow, respectively. For small Prandtl number liquids ($Pr \leq 0.01$), the perturbation energy mainly comes from the interaction between the perturbation and the basic flow, which suggests that the instability mechanism is hydrodynamic. The interaction between the perturbation and the basic flow in the azimuthal direction becomes negative when a moderate rotation is applied on the disks, and the moderate rotation can stabilize the thermocapillary flows for small Prandtl number liquids.

1. Introduction

Thermocapillary flows are important in the floating-zone, Czochralski crystal growth [1] and microgravity materials processing [2]. The phenomena arisen from thermocapillary flows could interact with other transport and phase-change phenomena. Campbell et al. [3] studied the macrosegregation in the crystals grown with the floating-zone technique. A germanium seed with a diameter of 8 mm was installed inside an ampoule. When seed rotation changed from 0 to 2 rpm, a striation pattern appeared in the crystal. In this growth process, there exist melting and solidifying interfaces at the top and bottom of melt. To investigate the mechanism of the oscillatory convection which may cause striations in the crystals grown by the floating-zone technique, liquid bridge is usually used to imitate half of the floating zone [1]. Many researchers performed numerical simulations on the thermocapillary flows in liquid bridges. For crystal-growth applications, the melts are usually liquids with $Pr \ll 1$. The Prandtl numbers for germanium, silicon, gallium arsenide are 0.008, 0.027, 0.068, respectively [4]. For small Prandtl numbers ($Pr \ll 1$) the first instability of the axisymmetric flow is a stationary bifurcation which is considered hydrodynamic in nature [5–9]. For liquid bridges with Prandtl numbers $Pr \geq 1$, the first instability is oscillatory which is analogous to the hydrothermal-wave instability [5,10–12]. There are two different oscillatory instabilities for high Prandtl number silicone oils ($Pr = 30$ and 74): a pulsating standing wave and a rotating traveling wave [13]. In the experiments, the onset oscillation can be either a traveling wave or a pulsating standing wave, depending on the perturbation introduced at the beginning of the experiments. The pulsating standing wave results from the superposition of two counter-propagating azimuthal hydrothermal waves with equal amplitude.

In production of crystals by the floating zone method, the feed and crystal rods are often rotating in order to suppress the azimuthal asymmetry. A model of liquid bridge with counter-rotating disks can be proposed to study stability phenomena in a half part of the zones (Fig. 1). Here, the liquid bridge is formed between two disks of diameter $2R_0$, which are separated by a distance L. Two temperatures T_L and $T_L + \Delta T$ are applied to the lower and upper disks. We assume that the two coaxial disks are counter-rotating with the same angular velocity.

The purpose of this paper is to study instability mechanism of the thermocapillary flows between counter-rotating disks by linear stability analysis. The impacts of Prandtl number, the aspect ratio

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https://doi.org/10.1016/j.ijheatmasstransfer.2017.09.137
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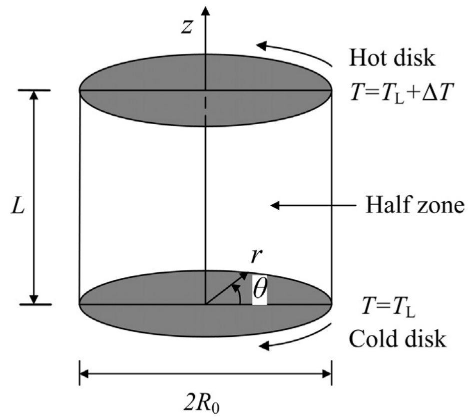

Fig. 1. Schematic of a liquid bridge between counter-rotating disks.

and the Coriolis number on the critical capillary Reynolds number are investigated. In Section 2, the physical and mathematical models and numerical methods are described. In Section 3, we calculate the critical capillary Reynolds numbers and critical frequencies for different Coriolis numbers.

2. Problem formulation

Typical dimensionless parameters such as aspect ratio, Marangoni number, Prandtl number, capillary Reynolds number and Coriolis number are defined as,

$$A = \frac{L}{2R_0}, \quad Ma = \frac{U_0 R_0}{\alpha}, \quad Pr = \frac{\nu}{\alpha}, \quad Re_\gamma = \frac{Ma}{Pr}, \quad \tau = \Omega R_0^2/\nu, \tag{1}$$

where the reference velocity is $U_0 = |\sigma'_T|\Delta T/\rho_0 \nu$, and ρ_0, ν, α, σ'_T denote liquid density, kinematic viscosity, thermal diffusivity, surface tension derivative with respect to temperature, respectively. Ω is the angular velocity of the upper disk, which is positive when rotating counter-clockwise viewing from the top.

The dimensionless governing equations and boundary conditions for the basic state fields can be obtained as in Ref. [14]. In linear stability analysis of thermocapillary convections, the small amplitude fluctuations of the velocities $\mathbf{u} = (u, v, w)$, pressure p and temperature T are imposed on the basic state. The dimensionless governing equations for the perturbation quantities can be obtained as [11,14],

$$\nabla \cdot \mathbf{u} = 0, \tag{2a}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}_0 + \mathbf{u}_0 \cdot \nabla \mathbf{u} = -\nabla p + \nabla^2 \mathbf{u}, \tag{2b}$$

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T_0 + \mathbf{u}_0 \cdot \nabla T = \frac{1}{Pr}\nabla^2 T. \tag{2c}$$

After applying the curl operator to the momentum equation (2b) to eliminate the pressure term, we can obtain the vorticity equations. For solving the eigenvalue problem, the continuity equation (2a), the vorticity equations in r and z directions, and the energy equation (2c) are used for $m > 0$, while the continuity equation, the vorticity equation in the azimuthal direction, and the energy equation are used for $m = 0$. Here, m denotes the azimuthal wavenumber of perturbation. The continuity equation is included in the perturbation equations to ensure that the continuity equation is satisfied at all grid points. The boundary conditions for the perturbation equations are set as in the following.

The boundary conditions at the disks ($z = 0$ and $z = 2A$) are given by,

$$\mathbf{u} = 0 \text{ and } T = 0. \tag{3a}$$

For $m = 0$, the continuity equations $\frac{\partial w}{\partial z} = 0$ at $z = 0$ and $z = 2A$ are included in the perturbation equations, and the vorticity equations in the azimuthal direction at grid points adjacent to the solid disks are not included. The boundary conditions at the free surface are,

$$\mathbf{u} \cdot \mathbf{n} = 0, \quad \mathbf{t} \cdot \mathbf{S} \cdot \mathbf{n} = -\frac{Ma}{Pr}\mathbf{t} \cdot \nabla T,$$
$$\mathbf{s} \cdot \mathbf{S} \cdot \mathbf{n} = -\frac{Ma}{Pr}\mathbf{s} \cdot \nabla T, \quad \text{and} \quad \mathbf{n} \cdot \nabla T = 0, \tag{3b}$$

where $\mathbf{s}$ denotes the tangential unit vectors in the horizontal cross-sections. The conditions at the central axis, $r = 0$, are taken as [11],

$$u = 0, \quad \frac{\partial w}{\partial r} = 0, \quad \frac{\partial T}{\partial r} = 0, \quad \text{for} \quad m = 0,$$

$$\frac{\partial u}{\partial r} = 0, \quad u + \frac{\partial v}{\partial \theta} = 0, \quad w = 0, \quad T = 0, \quad \text{for} \quad m = 1,$$

$$u = 0, \quad v = 0, \quad w = 0, \quad T = 0, \quad \text{for} \quad m > 1. \tag{3c}$$

The perturbation quantities (u, v, w, p, T) can be expanded as a sum of normal modes,

$$\begin{pmatrix} u \\ v \\ w \\ p \\ T \end{pmatrix} = \sum_m e^{\sigma t + im\theta} \begin{pmatrix} \tilde{u}_m(r,z) \\ im\tilde{v}_m(r,z) \\ \tilde{w}_m(r,z) \\ \tilde{p}_m(r,z) \\ \tilde{T}_m(r,z) \end{pmatrix} + \text{c.c.}, \tag{4}$$

where $\sigma = \sigma_r + i\sigma_i$, σ_r and σ_i are the growth rate and frequency of small perturbation, respectively, i denotes the complex unit $\sqrt{-1}$, and c.c. denotes the complex conjugate.

The basic state and perturbation equations are solved using the Chebyshev-collocation method. For calculation of the basic-state flow and temperature fields, the numerical computations are based on a pseudo-spectral Chebyshev method [15]. For cases with large capillary Reynolds numbers, we use 49×129 Chebyshev polynomials in r and z directions. The dimensionless time step for the basic state calculation is set as $\Delta t = 10^{-7}$. For Calculation of the perturbations, 37×97 Chebyshev polynomials are used. The eigenvalues and eigenfunctions are then obtained using the Q-R method. Since four perturbation equations are used, the complex matrix for the generalized eigenvalue problem has a size of (4MN) [2] with $M = 37$ and $N = 97$. Linear interpolations for both Re_{γ_c} and σ_{i_c} are adopted

3. Results

3.1. Dependence of the critical capillary Reynolds number on the aspect ratio

Fig. 2a and b shows dependence of the critical capillary Reynolds number Re_{γ_c} on the aspect ratio A for liquid bridges with $Pr = 0.01$ and $Pr = 0.001$, respectively. The most unstable modes are indicated near symbols in the figures, except those which have the mode $m = 1$. There are still some differences of features between the modes for different Coriolis numbers. When no rotation is applied, the region of the mode $m = 2$ is $A = 0.5$–0.8 for both $Pr = 0.01$ and 0.001. When the Coriolis number $\tau = 50$, the region of the mode $m = 2$ is only at $A = 0.5$ for both $Pr = 0.01$ and 0.001. When $\tau = 100$, the region of the mode $m = 2$ ranges from $A = 0.8$ to 1.05 for both $Pr = 0.01$ and 0.001.

The critical capillary Reynolds numbers for $Pr = 0.001$ are generally smaller than those for $Pr = 0.01$, and the differences are about 4.9–9.7% of the values for $Pr = 0.01$. Instabilities are caused by the interaction between the perturbation and the basic flow for $Pr \leq 0.01$. However, the dissipating effect of the thermocapillary force