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Vibration effect on double-diffusive instability in an inhomogeneous porous layer underlying a binary fluid layer

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ABSTRACT

A linear stability problem for mechanical equilibrium in an inhomogeneous fluid-saturated porous layer underlying a horizontal binary fluid layer is numerically simulated. The layers are subjected to high-frequency vertical vibration in a gravity field. Different values of temperature and concentration are fixed at the outer impermeable boundaries of the two-layer system. Porosity of a porous layer linearly depends on a transverse *z*-coordinate. Permeability is given by the Carman-Kozeny formula. To describe a fluid flow in layers, we use the vibrational convection equations written in the Boussinesq approximation and obtained by the averaging method. It is shown that when the fluid is heated from below, vibration effectively increases the equilibrium stability threshold and wavelength of its most dangerous perturbations. If a temperature gradient coincides with the direction of a concentration gradient for the heavier component of binary fluid, average convection is excited in an oscillatory manner. In the opposite case, there is a monotonic instability of equilibrium. Effects of the dimensionless porosity gradient m_z and vibrational parameter p_v on the instability threshold are studied. For $p_v < 0.0134$ and a fixed solutal Rayleigh number ($-5 \leqslant R_{mc} \leqslant 15$) an abrupt jump-like transition from the long-wave to short-wave most dangerous perturbations occurs as m_z grows in the range of 0.030–0.162. Long-wave perturbations penetrate both layers. Short-wave perturbations mainly locate in the fluid layer. The transition is smoothed for $p_v > 0.0134$. When the fluid is heated from above, wavelength of critical perturbations smoothly varies with a change in p_v and m_z. A convective fluid flow arises monotonously and mostly in the form of long wavelength rolls (a wave number is of $k < 3.3$). Vibration weakly lowers the equilibrium stability threshold in layers. The effect is most pronounced at high enough Rayleigh numbers ($R_m = -40$) for a porous medium with the porosity increasing with depth at $m_z = -0.2$.

1. Introduction

Changes in fluid density with temperature cause thermal gravitational convection in a differentially heated porous layer saturated with a single-component fluid under a gravity field [1,2]. Mechanical equilibrium of the fluid in a porous layer with impermeable boundaries loses its stability at $R_m = 4\pi^2$ and $k = \pi$, where R_m is the minimal critical Rayleigh number and k is wave number of the most dangerous perturbations of equilibrium [2]. In a binary-fluid-saturated porous layer the instability is due to density variations with temperature and concentration. Double-diffusive convection can be excited either in a monotonic or oscillatory manner. It depends on thermal diffusion and mass diffusion properties of fluid-saturated porous medium and a relationship between the temperature and concentration gradients. Analytical expressions for the monotonic and oscillatory instability thresholds in a porous layer with impermeable boundaries that are at fixed different temperatures and concentrations were derived in [2,3].

When a fluid layer is partially filled with a porous medium, two layers form. A fluid layer has the relative thickness $d = h_f/h_m$, where h_f is a fluid layer thickness and h_m is a porous layer thickness. A peculiarity of convection excitation in the two-layer system is bimodal neutral curves of equilibrium stability [4–8]. For small values of d, convection occurs in the form of long wavelength rolls covering both layers. A sharp change in the instability nature is observed as d grows. Perturbations of short wavelength become the most dangerous at large enough values of d. They mainly locate in the fluid layer. The jump-like variation of critical perturbation

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wavelength was previously found in [9–11]. Convection in a three-layer system consisting of a single-component fluid layer surrounded by two homogeneous porous layers was investigated.

Since the fluid flowing through a porous medium experiences the resistance of porous matrix, a change in matrix properties (porosity and permeability) affects convective instability. The onset of convection in superposed fluid and porous layers with variable porosity and permeability in a gravity field was explored in [12–14]. An inhomogeneous porous layer was saturated with a single- or two-component fluid. It was shown that an increase in porosity with depth within the layer more effectively lowers the instability threshold with respect to long-wave perturbations penetrating into this layer. The variable porosity effect on short-wave instability is less pronounced.

A study of double-diffusive convection in superposed binary fluid and fluid-saturated porous layers has its application to the binary solutions or alloys directionally solidified in a gravity field [15–18]. When a solution is cooled and solidified from below, a directed upward concentration gradient of its heavier component occurs. The destabilizing concentration and stabilizing temperature gradients give rise double-diffusive convection in a gravity field. Convective fluid flows in a solution layer overlying a solution-saturated porous layer (mushy zone) that forms near the upper crystal boundary are caused by the growth of short-wave perturbations of the motionless state [18]. Long-wave perturbations penetrate both layers. Convection in the solution and adjacent mushy zone deforms a crystal. Short-wave instability is responsible for the formation of double-diffusive fingers near at the upper boundary of mushy zone. Long-wave instability results in plume convection and produces chimneys inside the mushy zone.

Vibration control convective heat and mass transfer in layers. An average flow of the fluid with an inhomogeneous density arises under gravity and high-frequency vibration. Changes in density of a single-component fluid heated from below can be related to a temperature gradient. The vibration effect on quasi-equilibrium in a fluid layer [19,20] or a fluid-saturated porous layer [21–25] depends on the orientation of vibration axis with respect to the acceleration of gravity and temperature gradient. Vibration can play both a stabilizing role and a destabilizing role in comparison with the case of static gravity field. Average fluid velocity is zero in the quasi-equilibrium state. Pulsation fluid velocity can be non-zero. Both fluid velocities are equal to zero in the mechanical equilibrium state. If the vibration axis, vertical temperature gradient and acceleration of gravity are co-directed, a stability threshold of mechanical equilibrium increases as vibration intensity enhances.

The averaging method is used to describe thermal vibrational convection in differentially heated fluids and fluid-saturated porous media in the presence of high-frequency and small-amplitude vibration [19–25]. Assuming that the vibration period is small relative to the reference hydrodynamic and thermal times, we can represent temperature, velocity, and pressure as sums of their components averaged over the vibration period and pulsation components oscillating with small amplitudes. Criteria for the onset of average convection are determined.

The vibration effect on convective instability is equivalent to the periodic gravity modulation effect. In the case of finite frequency vibration, periodic solutions of the Mathieu equation are found. Frequency intervals are considered not only for synchronous periodic solutions but also for subharmonic solutions with a double vibration period. A direct method of linear stability analysis was applied in [26–29] for a differentially heated fluid-saturated porous layer subjected to vertical vibration. It was noted that vibration slowly destabilizes the mechanical equilibrium with respect to subharmonic perturbations in a single-component fluid heated from below. A stability threshold relative to synchronous perturbations goes up as vibration intensity increases. A transition from synchronous to subharmonic oscillations is revealed at a certain value of the dimensionless vibration frequency [28].

The onset of average double-diffusive convection in a rectangular porous cavity saturated with a binary fluid and subjected to high-frequency vibration in a gravity field was investigated in [30,31]. It was found that vertical vibration can destabilizes the mechanical equilibrium state in some range of parameters that depend on a relation between thermal diffusion and mass diffusion times and a ratio of temperature and concentration gradients. Analytical formulas are derived for the monotonic and oscillatory instability thresholds.

Works [32,33] are devoted to the high-frequency vibration effect on quasi-equilibrium stability of a differentially heated binary fluid with the Soret effect in a rectangular porous cavity. Transversal vertical vibration suppresses average convection as it is in the case of a single-component fluid. Longitudinal vibration strengthens convection. The authors of [34,35] studied the vertical high- or moderate-frequency vibration effect on Soret-driven convection in a binary fluid layer. High-frequency vibration raises the instability threshold. In the case of finite vibration frequencies, parametric convection excitation is possible. There are frequency intervals typical for subharmonic and synchronous as well as for quasi-periodic perturbations of equilibrium. Both stabilization and destabilization of fluid equilibrium can be observed with increasing the vibration intensity. Quasi-periodic instability occurs in a differentially heated binary fluid under modulated gravity field due to the growth of perturbations with two incomparable frequencies: frequency of natural oscillatory perturbations (eigenfrequency) and frequency of an external vibrational field. The onset of convection and dynamic of nonlinear convective flows corresponding to quasi-periodic oscillations was analyzed in [36,37]. It was found that alternating action gradually decreases the intensity of quasi-periodic oscillatory flow and even suppresses it in some range of vibration amplitudes.

Thermal convection in a heated from below fluid-saturated porous layer underlying a horizontal single-component fluid layer under vertical vibration in a gravity field was considered in [38–42]. It was shown that convection monotonically arises. High-frequency vibration plays a stabilizing role. It increases wavelength of the most dangerous perturbations of mechanical equilibrium. For some range of parameters (the Darcy number, ratio of layer thicknesses, etc.) an abrupt jump-like transition from the short-wave to long-wave instabilities occurs as vibration intensity grows. Vibration effectively suppresses short-wave instability in the fluid layer, where inertial effects are more pronounced than in the porous layer. The equilibrium stability threshold relative to long-wave perturbations covering both layers slightly rises as the vibrational parameter runs up. For superposed single-component fluid and porous layers with variable porosity a sharp jump-like change in the critical perturbation wavelength is smoothed at sufficiently high vibration intensities [43].

Excitation of average double-diffusive convection in a fluid layer overlying a porous layer with variable porosity under high-frequency vertical vibration in a gravity field was studied in [44]. Porosity linearly varies with depth as in the two-layer system considered in [43], but the layers were filled with a binary fluid. The temperature and concentration gradients were constant and vertical in a mechanical equilibrium state. The authors restricted their analysis to monotonic instability of equilibrium. They obtained stability maps $R_{m\,\min}(p_v)$ at a fixed solutal Rayleigh number $R_{mc} = -10$ for the case of heating from below and $R_{mc\,\min}(p_v)$ at a fixed Rayleigh number $R_m = -10$ for the case of heating from above. Here p_v is the vibrational parameter. The dimensionless porosity gradient m_z had values $-0.2, 0, 0.2$. It was shown that vibration rises the