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Analyzing natural convection in porous enclosure with polynomial chaos expansions: Effect of thermal dispersion, anisotropic permeability and heterogeneity

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ABSTRACT

In this paper, global sensitivity analysis (GSA) and uncertainty quantification (UQ) have been applied to the problem of natural convection (NC) in a porous square cavity. This problem is widely used to provide physical insights into the processes of fluid flow and heat transfer in porous media. It introduces however several parameters whose values are usually uncertain. We herein explore the effect of the imperfect knowledge of the system parameters and their variability on the model quantities of interest (QoIs) characterizing the NC mechanisms. To this end, we use GSA in conjunction with the polynomial chaos expansion (PCE) methodology. In particular, GSA is performed using Sobol' sensitivity indices. Moreover, the probability distributions of the QoIs assessing the flow and heat transfer are obtained by performing UQ using PCE as a surrogate of the original computational model. The results demonstrate that the temperature distribution is mainly controlled by the longitudinal thermal dispersion coefficient. The variability of the average Nusselt number is controlled by the Rayleigh number and transverse dispersion coefficient. The velocity field is mainly sensitive to the Rayleigh number and permeability anisotropy ratio. The heterogeneity slightly affects the heat transfer in the cavity and has a major effect on the flow patterns. The methodology presented in this work allows performing in-depth analyses in order to provide relevant information for the interpretation of a NC problem in porous media at low computational costs.

1. Introduction

Natural convection (NC) in porous media can take place over a large range of scales that may go from fraction of centimeters in fuel cells to several kilometers in geological strata [1]. This phenomenon is related to the dependence of the saturating fluid density on the temperature and/or compositional variations. A comprehensive bibliography about natural convection due to thermal causes can be found in the textbooks and handbooks by Nield and Bejan [1], Ingham and Pop [2], Vafai [3] and Vadasz [4]. Comprehensive reviews on NC due to compositional effects have been provided by Diersch and Kolditz [5], Simmons et al. [6], Simmons [7] and Simmons et al. [8]. NC in porous media can be encountered in a multitude of technological and industrial applications such as building thermal insulation, heating and cooling processes in solid oxide fuel cells, fibrous insulation, grain storage, nuclear energy systems, catalytic reactors, solar power collectors, regenerative heat exchangers, and thermal energy storage [1,2,9]. Important applications can be also found in hydro-geology and environmental fields such as in geothermal energy [10,11], enhanced recovery of petroleum reservoirs [12–14], geologic carbon sequestration [15–19], saltwater intrusion in coastal aquifers [20] and infiltration of dense leachate from underground waste disposal [21].

Numerical simulation has emerged as a key approach to tackle the aforementioned applications in the last two decades. This is today a powerful and irreplaceable tool for understanding and predicting the behavior of complex physical systems. The literature concerning the numerical modeling and simulation of convective flow in porous media is abundant [22–27]. The NC in porous media is usually described by the conservation equations of fluid mass, linear momentum and energy, respectively. Either Darcy or Brinkman models are used as linear momentum conservation laws. Darcy model is a simplification of the Brinkman model which

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neglects the effect of viscosity. This simplification is valid for low permeable porous media. For high permeable porous media Brinkman model is more suitable because the effective viscosity is about 10 times the fluid viscosity [28–30]. In the traditional modeling analysis of NC in porous media, the governing equations are solved under the assumption that all the parameters are known. However, in real applications, the determination of the input parameters may be difficult or inaccurate. For instance, in the simulation of geothermal reservoirs, the physical parameters (i.e. hydraulic conductivity and porosity) are subject to significant uncertainty because they are usually obtained by model calibration procedures, that are often carried out with relatively insufficient historical data [31].

The uncertainties affecting the model inputs may have major effects on the model outputs. Typical examples about the significance of these effects (that are not exhaustive) can be found in the design of clinical devices or biomedical applications where small overheating can lead to unexpected serious disasters [32–34]. Hence, the evaluation of how the uncertainty in the model inputs propagates and leads to uncertainties in the model outputs is an essential issue in numerical modeling. In this context, uncertainty quantification (UQ) has become a must in all branches of science and engineering [35–37]. It provides a rigorous framework for dealing with the parametric uncertainties. In addition, one wants to quantify how the uncertainty in the model outputs is due to the variance of each model input. This kind of studies is usually known as sensitivity analysis (SA) [38]. UQ aims at quantifying the variability of a given response of interest as a function of uncertain input parameters, whereas GSA allows to determine the key parameters responsible for this variability. UQ and GSA are usually conducted by a multi-step analysis. The first step consists on the identification of model inputs that are uncertain and modeling them in a probabilistic context by means of statistical methods using data from experiments, legacy data or expert judgment. The second step consists in propagating the uncertainty in the input through the model. Finally, sensitivity analysis is carried out by ranking the input parameters according to their impact onto the prediction uncertainty. UQ and GSA have proven to be a powerful approach to assess the applicability of a model, for fully understanding the complex processes, designing, risk assessment and decision-making. They have been extensively investigated in the literature (e.g. [39–50]). In the frame of flow and mass transfer in porous media, UQ and GSA have been applied to problems dealing with saturated/unsaturated flow [51], solute transport [52,53] and seawater intrusion in coastal aquifer [49,54,55]. Recently, Shao et al. [56] developed a new technique for GSA in the Bayesian framework based on the Kashyap information criterion. The new strategy has been validated on a simplified double-diffusive convection problem. A careful literature review shows that the investigation of sensitivity analysis for NC in porous media has been limited to some special applications [48]. To the best of our knowledge, UQ and GSA have never been performed for a problem involving NC within a porous enclosure. Yet, NC in porous enclosure has been largely investigated for several purposes [57,58] and several authors have contributed important results for such a configuration [57,59–70]. Hence, keeping in view the various applications of NC in porous enclosure and the importance of uncertainty analysis in numerical modeling, a complete analysis involving GSA and UQ study is developed in this work to address this gap. The considered problem deals with the square porous cavity. Such a problem is widely used as a benchmark for numerical code validation due to the simplicity of the boundary conditions [30,65,71–77]. It is also widely used to provide physical insights and better understanding of NC processes in porous media [65,66,78–82]. As model inputs, we consider the physical parameters characterizing the porous media and the saturating fluid as the permeability, porosity, thermal diffusivity and thermal expansion. All these parameters can be described by the Rayleigh number which represents the ratio between the buoyancy and diffusion effects.

A common simplification for NC in porous media is to consider the saturated porous media with an equivalent thermal diffusivity (based on the porosity) and neglect the key process of heat mixing related to velocity dependent dispersion. Yet several studies have found that thermal dispersion plays an important role in NC systems [83–95] and applications related to transport in natural porous media [96–98]. Hence, main attention is given here to understand the impact of anisotropic thermal dispersion by including the longitudinal and transverse dispersion coefficients in the model inputs. Furthermore, anisotropy in the hydraulic conductivity is acknowledged as it is one of the properties of porous media which is a consequence of asymmetric geometry and preferential orientation of the solid grains [76]. Finally heterogeneity of the porous media is considered as a source of uncertainty as it has a significant impact on NC in porous media [6,77,99,100]. As model outputs, we consider different quantities that are often used to assess the flow and the heat transfer processes in porous cavity as the temperature spatial distribution, the Nusselt number and the maximum velocity components.

In this work, we perform a global sensitivity analysis using a variance-based technique. In this particular context, the Sobol sensitivity indices [101–103] are widely used as sensitivity metrics, because they do not rely on any assumption regarding the linearity or monotonous behavior of the physical model. Various techniques have been proposed in the literature for computing the Sobol indices, see e.g. [38,103–106]. Monte Carlo (MC) is one of the most commonly used methods. However, it might become impractical, because of the large number of repeated simulations required to attain statistical convergence of the solution, especially for complex problem (e.g., [40,107] and references therein). In this context, new approaches based on advanced sampling strategies have been introduced to reduce the computational burden associated with Monte Carlo simulations. Among different alternatives, Polynomial Chaos Expansions (PCE) have been shown to be an efficient method for UQ and GSA [108–110]. In PCE, the key idea is to expand the model response in terms of a set of orthonormal multivariate polynomials orthogonal with respect to a suitable probability measure [111]. They allow one to uncover the relationship between different input parameters and how they affect the model outputs. Once a PCE representation is available, the Sobol' sensitivity indices are then obtained via a straightforward post-processing analysis without any additional computational cost [40]. It can also be used to perform an uncertainty quantification using Monte Carlo analysis at a significantly reduced computational cost (see, e.g., [52] and references therein).

The structure of the present study is as follows. Section 2 is devoted to the description of the benchmark problem and the governing equations. Section 3 describes the numerical model. Section 4 describes the sensitivity analysis procedure using Sparse PCE. Section 5 discusses the GSA and UQ results for homogeneous and heterogeneous porous media. Finally, a summary and conclusions are given in Section 6.

2. Problem statement and mathematical model

The system under consideration is a square porous enclosure of length H filled by a saturated heterogeneous porous medium. The properties of the fluid and the porous medium are assumed to be independent on the temperature. The porous medium and the saturating fluid are locally in thermal equilibrium. We assume that the Darcy and Boussinesq approximations are valid and that the inertia and the viscous drag effects are negligible. Under these