International Journal of Heat and Mass Transfer 115 (2017) 225–231

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Fractal analysis of flow resistance in random porous media based on the staggered pore-throat model

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ARTICLE INFO

Article history:
Received 23 November 2016
Received in revised form 4 July 2017
Accepted 6 July 2017

Keywords:
Porous media
Flow resistance
Staggered pore-throat model
Fractal

ABSTRACT

Flow resistance in porous media is a hot and difficult problem due to its importance and the complexity of the fluid flow in porous media. To improve the aligned pore-throat model, a more reasonable staggered pore-throat model is established. Fractal is employed in random porous media and the analytical flow resistance formulation is derived. The formulation is the function of tortuosity, porosity, pore-throat ratio, interference coefficient and so on. It's found that the staggered pore-throat model and the original pore-throat model have their own applicable ranges of Reynolds number and porosity respectively. The interference coefficient is negligible at low Reynolds number, while considerable at high Reynolds number. The value of the interference coefficient c is determined to be 0.8.

1. Introduction

The study of the fluid flow in porous media has wide engineering application [1] such as groundwater seepage, oil and gas extraction, thermal insulation materials, pebble-bed cooling reactor, porous material drying and biotechnology. Therefore, it is important to study the flow resistance of porous media. Lots of flow resistance formulas are obtained empirically or semi empirically so far. Table 1 lists some main formulas.

In fact, the origin of the flow resistance in porous media can date back to Darcy's Law [16] in 1856. Darcy's Law shows that the pressure drop is linear with the velocity When $Re < 1$. The fluid suitable for Darcy's Law is called the Darcy flow. Forchheimer [17] pointed out that the pressure drop is characterized by the quadratic dependence of flow velocity When $Re \in [1\ 1\ 0]$. It's called Forchheimer flow. After that, Blake, Kozeny [33] and Carman [34] all proposed the correlation of flow resistance in porous media. In table 1, Ergun [3] summarized the previous studies and experimental data and proposed a comprehensive equation called Ergun equation. Ergun equation is the most widely used correlation so far. The influence of the fluid density, viscosity, porosity and particle size on the flow resistance is considered in the equation. The pressure drop is the sum of viscous loss and inertia loss of flow in the porous media. However, there are two empirical constants A and B in Ergun equation and their values are 150 and 1.75 respectively obtained by Ergun. They are not universal constants. Afterward the researchers derived different values of A and B listed in the Table 2, and some researcher thinks that A and B related to sphericity.

In the recent years, Yu Boming et al. [23–27] derived a reasonable fractal resistance correlation as Eq. (1) based on the aligned pore-throat model and fractal theory.

$$\frac{\Delta P}{L} = \frac{1}{\phi_s}\frac{32\mu}{L^{1-D_T}}\frac{1-\varepsilon}{\varepsilon}\frac{D_T - D_f + 3}{2 - D_f}\frac{1}{\lambda_{\max}^{D_T+1}}v_s + \frac{1}{\phi_s^2}\left(\frac{1}{\beta'^4} - \frac{5}{2\beta'^2} + \frac{3}{2}\right)\frac{\rho}{2\varepsilon^2}\frac{D_T^2}{l'}\left(\frac{L}{\bar{\lambda}'}\right)^{2D_T-2} v_s^2 \quad (1)$$

And Yu Boming is the first one to put forward such a creative idea and method, since fractal is used to study the fluid flow in porous media, and experiment data show that the result is better than Ergun equation's based on some literature data. The researcher also believe that more data is still needed to be used to verify it further. Zhang Lijuan et al. [28] prove that resistance characters of fluid flow in porous media is similar to that of core seepage by experiments. It is illustrated that pore-throat model formed by expansion and contraction flow channels is suitable. Yu Boming et al. believe that the pore-throat model is consisted of aligned packing particles. However, observing and analyzing from lots of packing bed, we believe

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.07.031
0017-9310/

Nomenclature

ρ	density, kg m^{-3}	L	length of REV, m
μ	dynamic viscosity, Pa s	$\bar{\tau}$	average tortuosity
ΔP	total pressure drop or resistance loss, Pa	v_s, u	superficial velocity, m/s
ΔP_1	friction resistance loss, Pa	N	numbers of pore or pipe
ΔP_2	local resistance loss, Pa	D_f	fractal dimension of pore
G	dimensionless pressure drop or resistance loss	D_T	fractal dimension of tortuous
$\Delta P/L$	total pressure drop of unit length, Pa	a	diameter of the pore, m
Re_p	particle Reynolds number	b	diameter of the inlet throat, m
ε	porosity	l	average length of the pore-throat, m
d_p	equivalent diameter of particle, m	β	pore-throat ratio
d	topological dimension	ζ_i	expansion local resistance coefficient
λ	pore or capillary diameter, m	ζ_o	sudden contraction local resistance coefficient
λ_{max}	minimum diameter of pore or capillary, m;	ζ	total local resistance coefficient
λ_{min}	maximum diameter of pore or capillary, m	$\bar{v}$	real mean velocity, m/s
$\bar{\lambda}$	average pore or capillary diameter, m	c	interference coefficient
L_t	tortuous length, m	$\bar{v}_t$	velocity in the tortuous capillary, m/s

Table 1
The main formulas of flow resistance in porous media.

Researcher	Resistance formulation	Range of validity
Rose [2], 1945	$\frac{\Delta P}{L} = (1000Re_p^{-1} + 60Re_p^{-0.5} + 12)\frac{\rho u^2}{d_p}$	Re_p very large
Rose and Rizk [3], 1949	$\frac{\Delta P}{L} = (1000Re_p^{-1} + 125Re_p^{-0.5} + 14)\frac{\rho u^2}{d_p}$	Re_p very large
Ergun [4], 1952	$\frac{\Delta P}{L} = 150\frac{(1-\varepsilon)^2}{\varepsilon^3 d_p^2}\mu u + 1.75\frac{(1-\varepsilon)}{\varepsilon^3 d_p}\rho u^2$	$1 \le Re_p \le 2000$
Kuerten [5], reported by Watanabe [6], 1966	$\frac{\Delta P}{L} = \left[\frac{25}{4\varepsilon^3}(1-\varepsilon^2)\right](21Re_p^{-1} + 6\mathrm{Re}_p^{-0.5} + 0.28)\frac{\rho u^2}{d_p}$	$0.1 \le Re_p \le 4000$
Aerov and Tojec [7], 1968	$\frac{\Delta P}{L} = \left(\frac{36.4}{Re_p} + 0.45\right)\frac{3(1-\varepsilon)\rho u^2}{\varepsilon^3 d_p}$	$Re < 2000$
Tallmadge [8], 1970	$\frac{\Delta P}{L} = \left(\frac{150}{Re_p} + \frac{4.2}{Re_p^{1/6}}\right)\frac{(1-\varepsilon)\rho u^2}{\varepsilon^3 d_p}$	$0.35 \le \varepsilon \le 0.88$ $10^{-1} \le Re_p \le 10^5$
Lee and Ogawa [9], 1974	$\frac{\Delta P}{L} = \frac{1}{2}\left[\frac{12.5}{\varepsilon^3}(1-\varepsilon^2)\right](29.32Re_p^1 + 1.56Re_p^{-n} + 0.1)\frac{\rho u^2}{d_p}\ n = 0.352 + 0.1\varepsilon + 0.275\varepsilon^2$	$1 \le Re_p \le 105$
Comiti and Renaud [10], 1989	$\frac{\Delta P}{L} = 2\mu\frac{(1-\varepsilon)^2}{\varepsilon^3}a^2\tau^2 u + 0.0986\tau^3 a\rho\frac{(1-\varepsilon)}{\varepsilon^3}u^2$	Laminar and turbulent flow
Seguin et al. [11], 1998	$\frac{\Delta P}{L} = \frac{\mu^2(1-\varepsilon)^3 a^3\tau}{2\rho\varepsilon^3}(Re_p + 0.0121Re_p^2)$	Laminar and turbulent flow
Hicks [12], 2002	$\frac{\Delta P}{L} = 6.8\frac{(1-\varepsilon)^{1.2}}{\varepsilon^3}Re_p^{-0.2}\frac{\rho u^2}{d_p}$	$500 \le Re_p \le 60000$
Yan X [13], 2006	$\frac{\Delta P}{L} = \left(\frac{117.9}{Re_p} + 0.63\right)\frac{3(1-\varepsilon)\rho u^2}{\varepsilon^3 d_p}$	$Re < 800$
Jamialahmadi and Müller-Steinhagen [14], 2005	$\frac{\Delta P}{L} = (\frac{25}{Re} + 0.292)\frac{6(1-\varepsilon)}{d_p\varepsilon^3}\rho u^2$	$Re < 10$
Montillet et al. [32], 2007	$\frac{\Delta P}{L} = a\left(\frac{1-\varepsilon}{\varepsilon^3}\right)\left(\frac{D}{d_p}\right)^{0.20}[1000Re_p^{-1} + 60Re_p^{-0.5} + 12]\frac{\rho u^2}{d_p}$	$10 < Re_p < 2500$ $a = 0.061$
Wu Jinsuiand Yu Boming et al. [15], 2008	$\frac{\Delta P}{L} = 72\frac{(1-\varepsilon)^2}{\varepsilon^3 d_p^2}\tau\mu u + \frac{3\tau}{4}\left(\frac{3}{2} + \frac{1}{\beta^4} - \frac{5}{2\beta^2}\right)\frac{(1-\varepsilon)}{\varepsilon^3 d_p}\rho u^2$	

Table 2
The values of *A* and *B* in Ergun equation.

Researcher	Forchheimer flow	Turbulent flow
Ergun [3]	$A = 150$, $B = 1.75$	
Wang Y B et al. [18]	$A = 411\Phi_s - 145$, $B = 2.8\Phi_s + 0.00698$	
Li Z P et al. [19]	$A = 160$, $B = 1.35$	$A = 193$, $B = 1.22$
Kececioglu [20]	$A = 180$, $B = 9.4(3 \le Re_p \le 20)$	$A = 342$, $B = 2.95(Re_p \ge 30)$
Fand [7]	$A = 182$, $B = 1.92$ $(5 \le Re_p \le 80)$	$A = 225$, $B = 1.61$ $(Re_p \ge 120)$
Irmay [21]	$A = 180$, $B = 0.6$	
Macdonald [22]	$A = 180$, $B = 1.8$	
Comiti et al. [10]	$A = 141$, $B = 1.63$	

staggered packing more reasonable in fact. Therefore, the staggered pore-throat model is established by modifying the original pore-throat model. A new flow resistance formula of porous media is derived from the fractal theory.

2. Fractal flow resistance analysis based on the staggered pore-throat model

The flow resistance in porous media is divided into two parts: the friction resistance and local resistance according to traditional fluid dynamics. The flow resistance of total is the sum of the two parts. The particle distribution, pore distribution, channel distribution, particle shape, tortuosity of channel etc. is fractal according to the fractal theory of porous media. Then fractal dimension can be used to describe their distribution characteristics. The fractal theory of porous media can be found in the relevant literatures [23–27].

2.1. Fractal formulation of friction resistance

Yu Boming and Wu Jinsui et al. derive the fractal formulation of friction resistance (Eq. (2)). We do not make changes and use it directly. The idea is that the channels in porous media are regarded