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A method for predicting thermal waves in dual-phase-lag heat conduction

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ABSTRACT

We predict whether thermal waves exist or not in dual-phase-lag heat conduction by measuring the time lag ratio (τ_q/τ_T). In this method, a one-dimensional model of cylindrical heating is developed and solved by the method of Laplace transform. The time lag ratio is then derived from the time derivative of the square of excess temperature on the heater wall in the initial heating stage. Experimentally, we also applied this method to measure the time lag ratios for sand and lean pork, both of which are less than 1 within acceptable measurement uncertainty, indicating no thermal waves generated. This method is of great importance for the application of dual-phase-lag heat conduction model since it can predict existence of thermal waves through a simple experiment.

1. Introduction

The classical heat conduction is described by Fourier's law. It assumes that heat flux and temperature gradient generate at the same time instant, which implies the heat disturbance propagates at an infinite speed [1]. Although this assumption is physically unacceptable, Fourier's law provides accurate results for most engineering problems at classical length and time scale [2,3]. However, experimental results are inconsistent with Fourier's law in extreme situations when the temperature is extremely low for instance approaching to absolute zero, or the heat flux is very high, or the heating period is very short or the heat conduction scale is extremely small, due to violation of the assumption of infinite heat propagation speed [4–7]. In addition, multiphase systems are reported to have non-Fourier heat conduction behavior as well, such as in nanofluids, porous media, and biomaterials [8–16].

To tackle the deviations from Fourier's law, Tzou [17–19] proposed the dual-phase-lag heat conduction model applicable to the heat conduction with high heat flux and short heating period, which is given by

$$q(t+\tau_q) = -k\nabla T(t+\tau_T) \tag{1}$$

where q is heat flux, T is temperature, τ_q and τ_T are the time lags of heat flux and temperature gradient respectively and k is thermal conductivity. In dual-phase-lag heat conduction, the time lag of heat flux (τ_q) indicates the effect of time scale which describes the thermal inertia in short time response, while the time lag of temperature gradient (τ_T) denotes the effect of length scale which describes delayed time caused by heat transport mechanism in microscale [20]. Different from Fourier's law, heat flux and temperature gradient are not synchronous in this model. Heat flux could have a time delay relative to the temperature gradient ($\tau_q > \tau_T$) and vice versa. Combined with energy conservation and taking the first-order approximation of Taylor expansion around t, Eq. (1) leads to,

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} + \frac{\tau_q}{\alpha}\frac{\partial^2 T}{\partial t^2} = \nabla^2 T + \tau_T\frac{\partial}{\partial t}(\nabla^2 T) \tag{2}$$

where α is the thermal diffusivity defined as $\alpha = k/\rho c$. If $\tau_q/\tau_T > 1$, Eq. (2) is hyperbolic and thermal waves will be produced in the heat conduction process. If, otherwise, $\tau_q/\tau_T \leqslant 1$, this equation is parabolic without thermal waves [21–23]. Wang et al. [1,24,25] applied Fourier's law to each individual phase of the multiphase systems such as nanofluids and porous media at microscale, and found that the macroscale heat conduction in each phase is exactly equivalent to dual-phase-lag heat conduction because of the cross coupling [26] of heat transfer in different phases. In addition, Liu and his coworkers' research indicated that dual-phase-lag thermal behavior also exists in bio-tissues, based on the inverse study of the experimental data [11,27,28]. As a result, Eq. (2) not only satisfies the heat conduction in extreme conditions, but is also an equivalent

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Nomenclature

Symbols

b	radius of cylindrical heater (m)
c	specific heat capacity (J kg^{-1} K^{-1})
k	thermal conductivity (W m^{-1} K^{-1})
K_0	zero-order modified Bessel function of the second kind
K_1	first-order modified Bessel function of the second kind
p	function of time lags
q	heat flux (W m^{-2})
$\bar{q}$	Laplace transform of heat flux
q_0	boundary heat flux (W m^{-2})
r	radial coordinate (m)
R^2	coefficient of determination
s	Laplace transform space
S	time derivative of fitted line ((°C)^2/s)
t	time (s)
T	temperature (°C)
T_0	initial temperature (°C)

Greek symbols

α	thermal diffusivity (m^2 s^{-1})
Δ	deviation of measurement
θ	excess temperature (°C)
$\bar{\theta}$	Laplace transform of excess temperature
ρ	density (kg m^{-3})
τ_q	time lag of heat flux (s)
τ_T	time lag of temperature gradient (s)
ν	order of modified Bessel function
Φ	function of time lags

description of heat conduction with multiphase coupling, such as in nanofluids, porous media and biomaterials [11,14,29].

When the time lag of heat flux (τ_q) is larger than that of temperature gradient (τ_T), thermal waves will be produced in the heat conduction process. In early days, thermal waves were reported to exist in extreme heat conductions such as high speed heating [30,31]. Then, Wang and Wei proved that thermal waves could exist in multiphase systems [24,25]. Thermal waves play an important role in the thermal behavior in dual-phase-lag heat conduction. For example, thermal wave could change contact thermal resistance and result in overshooting of temperature field in specific boundary condition [32–34]. Furthermore, thermal resonance could exist with an appropriate heat source or boundary condition in dual-phase-lag heat conduction with thermal waves [21]. The resonance will extremely enhance the heat transfer process in nanofluids and porous media. In addition, thermal waves also have a great influence on the temperature control in hyperthermia treatment and thermal damage of tissues [12,35]. Hence, it is vital to predict the existence of thermal waves. The measurement of the two time lags (τ_q and τ_T) is a direct approach to predict thermal waves. Tabrizi [36] suggested a method to measure τ_q and τ_T based on frequency response, but an essential parameter, medium frequency, is unknown in this method. Miranada [37] proposed a system to determine τ_q and τ_T, in which a semi-infinite layer is in thermal contact with a finite one and heated by a laser pulse. Tzou [20] proposed a possible approach to measure τ_q and τ_T through one-dimensional specimen via constant temperature gradient and constant heat flux respectively. Although several methods were proposed to measure the two time lags, few experiment studies have been conducted since these methods are difficult to carry out. Therefore, the experimental method to predict thermal waves is limited.

In this study, we propose an easily-implemented method to predict thermal waves in dual-phase-lag heat conduction by measuring the ratio between the two time lags (τ_q/τ_T). In this method, τ_q/τ_T is determined by the time derivative of the square of excess temperature on the heater wall in the initial heating stage. This method is then verified by the measurement in sand and lean pork. As the thermal waves could be predicted, this method is of great importance for the practical application of dual-phase-lag heat conduction model in extreme conditions and in porous media, nanofluids and biomaterials.

2. Theoretical analysis

In plane heating, it is difficult to determine the heat flux from the plane surface for measurement because of the heat loss from other surfaces. In addition, one dimensional plane heating model requires a relatively large surface area to eliminate the fringe effect. Therefore, a one-dimensional model of semi-infinite cylindrical heating is applied to measure the ratio between the two time lags (τ_q/τ_T), as shown in Fig. 1. In this model, the space is heated by a constant heat flux, q_0, which is released from the boundary of the cylindrical heater with radius of b. Meanwhile, it should be noted that the heat flux on the surface suddenly turns from 0 to q_0 ($q_0 > 0$) from $t = 0$.

According to Fig. 1, the energy conservation in cylindrical coordinates gives

$$-\left(\frac{1}{r}q + \frac{\partial q}{\partial r}\right) = \rho c\frac{\partial \theta}{\partial t}, (b, +\infty) \times (0, +\infty) \tag{3}$$

where θ is excess temperature ($\theta = T - T_0$, where T is transient temperature and T_0 is initial temperature equal to ambient temperature), ρ and c are the density and specific heat capacity of the

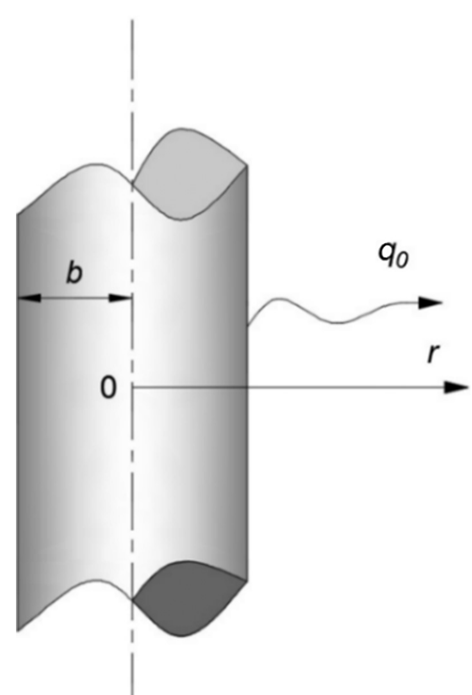

Fig. 1. Schematic of one-dimensional cylindrical heating. b is the radius of the heater and q_0 is the heat flux released from the boundary of the heater.