International Journal of Heat and Mass Transfer 115 (2017) 264–277

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

A comprehensive investigation of natural convection inside a partially differentially heated cavity with a thin fin using two-set lattice Boltzmann distribution functions

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ARTICLE INFO

Article history:
Received 24 May 2017
Received in revised form 6 July 2017
Accepted 10 July 2017

Keywords:
Heated cavity
Natural convection
Increasing Nusselt number
Small thin fin
Lattice Boltzmann method

ABSTRACT

Natural convection occurs in many engineering systems such as electronic cooling and solar collectors. Nusselt number (*Nu*) is one of the most important parameters in these systems that should be under control. This investigation is a comprehensive heat transfer analysis for partially differentially heated cavities with a small thin fin mounted on the hot wall of the cavity to increase or decrease the *Nu*. A Boussinesq approximation was utilized to model the buoyancy-driven flow. Two sets of distribution functions for the lattice Boltzmann method are used to resolve the continuity, momentum and energy equations. A total of 180 cases were analyzed to determine the average maximum and minimum *Nu* for the hot section of the cavity. For the cases investigated, the hot and cold sections were assumed to comprise approximately 30% of the vertical walls. After obtaining the velocity and temperature distributions for the cavity, three different Rayleigh numbers, i.e., 10^4, 10^5 and 10^6, were used to provide streamlines and isotherms for a number of cases. The average and normalized Nusselt numbers were then computed and compared graphically for all of the cases investigated. The results indicated that by choosing the appropriate position for the fin, the average *Nu* could be increased by more than 150%.

1. Introduction

Buoyancy driven natural convection in differentially heated rectangular cavities has been a fundamental problem of interest for many years and equates to a simplified representation of the thermal transport phenomenon occurring in a number of industrial applications, such as electronic cooling, solar collectors and nuclear reactors. Recently, relatively more complicated structures such as baffled L-shaped cavities [1], L-shaped enclosures with heating obstacle [2], and square cavities with internal and external heating [3] have been also considered. A number of fundamental studies have been performed to analyze and optimize the thermal performance of these structures. In addition, variations in the physical characteristics, such as the boundary conditions, Rayleigh number (*Ra*) and aspect ratio, have been investigated and evaluated. The results have indicated that there are two important subcategories into which these differentially heated cavities can be classified, partially heated/cooled vertical walls or internal fins in the cavities [4,5].

The addition of a fin inside an enclosure typically results in significant changes in the velocity and thermal fields within the system, which can often cause significant variations in the Nusselt number (*Nu*). This along with variations in the thermal characteristics and boundary conditions of the fin can impact the heat transfer aspects of the cavity [4,6]. Recently there have been several investigations of these phenomena and the effect of the variations cited above. Elatar et al. [6] analyzed the thermal performance of a square cavity with a thick fin attached to the vertical hot wall. A finite volume CFD approach, along with the Patanker solution algorithm was used to solve the problem. In these investigations, some interesting information was revealed such as the impact of variations on the fin effectiveness and the effect on average *Nu* as a result of variations in a number of parameters were analyzed and illustrated. Shi and Khodadadi [7] investigated the effect of a single thin fin mounted in various places on the horizontal or vertical walls, on the steady laminar flow and heat transfer within a lid-driven square cavity. After fluid flow and temperature distribution validations with previously published data [8,9], a detailed analysis regarding the average *Nu* of the cavity depending on the position of the fin was provided. Shi and khodadadi [4] extended the previous study [7] to a laminar natural convection heat transfer analysis in a differentially heated square cavity with a thin fin mounted on the

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.07.042
0017-9310/

Nomenclature

c	velocity vector
c_s	speed of sound
f^{eq}	equilibrium density distribution functions
g^{eq}	equilibrium energy distribution functions
g	acceleration of the gravity, m s^{-2}
H_f	non-dimensional height of the fin
H_h	non-dimensional height of the hot section
H_c	non-dimensional height of the cold section
h_f	height of the fin, m
h_h	height of the hot section, m
h_c	height of the cold section, m
L	width of the cavity, m
L_f	non-dimensional length of the fin
L_h	non-dimensional length of the hot section
L_c	non-dimensional length of the cold section
ℓ_f	length of the fin, m
ℓ_h	length of the hot section, m
ℓ_c	length of the cold section, m
Nu_ℓ	local Nusselt number
Nu_{ave}	average Nusselt number
Nu_{no}	normalized Nusselt number
p	pressure, Pa
P	non-dimensional pressure
Pr	Prandtl number
Ra	Rayleigh number
T	temperature, K
T_c	temperature of the cold plate, K
T_h	temperature of the hot plate, K
t	time, s
U, V	x–y non-dimensional velocity components
X, Y	non-dimensional Cartesian coordinates
u, υ	x–y velocity components (ms^{-1})
x, y	Cartesian coordinates
w	weighting factor

Greek symbols

θ	non-dimensional temperature
α	thermal diffusivity (m^2 s^{-1})
β	expansion coefficient (K^{-1})
ν	kinematic viscosity (m^2 s^{-1})
ρ	density of the fluid (kg m^{-3})
τ	non-dimensional time, relaxation factor

hot wall. Using a similar numerical approach the partial differential equations of the problem were solved. Local and average Nusselt numbers for both the hot and cold walls for a broad range of parametric values were determined and represented graphically. For both of these investigations [4,7], it was assumed that a perfectly conducting partition was mounted on a wall of the cavity. In a later investigation, Ben-Nakhi and Chamkha [10] conducted focused on the numerical study of steady, laminar, conjugate natural convection in a square enclosure, with an inclined thin fin of arbitrary length. The walls thicknesses of the cavity were also included in the simulation, and effects of the solid-to-fluid thermal conductivity ratio, thin fin inclination angle and length were examined to determine the effect on the local and average Nusselt numbers. Bilgen [11] also investigated a configuration similar to that studied in [2] using a finite thermal conductivity for the thin material and a

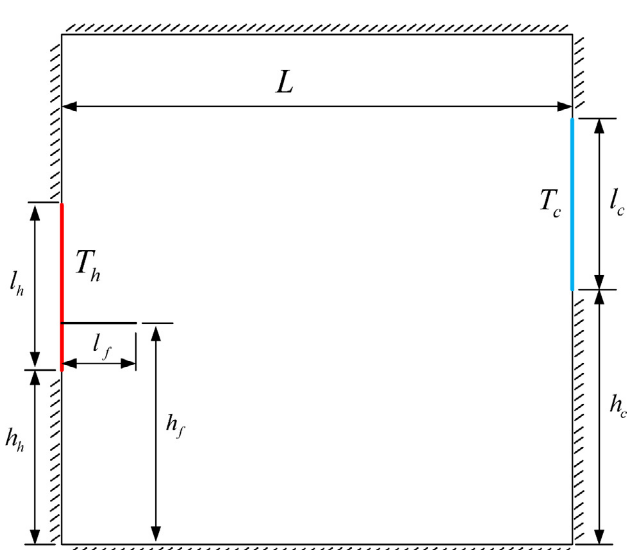

Fig. 1. Configuration of the partially differentially heated cavity.

range of Ra from 10^4 to 10^9. To reduce the heat transfer from the hot wall to the cold one, Ilis et al. [12] modified the previously investigated configurations [4,10,11] by mounting the thin fin on the ceiling of the cavity. Similar to previously studied cases, the local and total Nusselt numbers were examined in detail and the *Nu* ratio was plotted for different ranges of the parametric values. The results indicated that if a barrier is mounted on the top wall of the cavity, the heat transfer rate through the cavity can be significantly reduced. Liu et al. [13] numerically and experimentally investigated the natural convective flow adjacent to the finned sidewall of a differentially heated cavity. Comparison of the isotherms provided by a two-dimensional numerical analysis and the results obtained from a shadowgraph experimental investigation, provided considerable information on the flow structure around the fins and *Nu* of the heated side wall. Ma and Xu [14] also conducted a numerical investigation of the effect of fins on the thermal performance of cavities, using a geometry similar to that investigated by Liu et al. [13]. The numerical results were validated using the experimental shadowgraph results provided by Xu et al. [15].

Another category of heat transfer problems of interest is the one dealing with cavities that are partially heated and/or cooled. Sheremet et al. [16] performed a numerical analysis of MHD

Table 1
The average Nusselt number for all 27 cases without mounting a fin on the hot wall.

		$Ra = 10^4$	$Ra = 10^4$	$Ra = 10^4$
$H_h = 0.2$	$H_c = 0.2$	3.97675	6.93025	11.42202
	$H_c = 0.4$	4.35833	7.41139	12.28372
	$H_c = 0.6$	4.11400	7.06587	11.76870
$H_h = 0.4$	$H_c = 0.2$	3.75030	5.95782	8.17590
	$H_c = 0.4$	4.22751	7.11989	11.43304
	$H_c = 0.6$	4.08812	7.18521	12.18060
$H_h = 0.6$	$H_c = 0.2$	3.11733	4.46368	5.10298
	$H_c = 0.4$	3.54794	5.76873	8.27359
	$H_c = 0.6$	3.52977	6.37767	11.12218