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Dispersion for periodic electro-osmotic flow of Maxwell fluid through a microtube

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ABSTRACT

This paper mainly studies the solute dispersion driven by the periodic oscillatory electro-osmotic flow of viscoelastic fluid described by Maxwell constitutive modeling. Series expansion and transform methods are used to solve the unsteady convection dispersion equation. Analytic expressions of the natural dispersion coefficient $K(t)$ and the mean concentration C_m are derived. By numerical computation, the influences of several nondimensional parameters, such as electrokinetic width K, oscillating angular frequency ω of the external forced electrical field, oscillating Reynolds number *Re* and relaxation time *De* on the dispersion coefficient $K(t)$ and mean concentration C_m are investigated. The augment in amplitude of $K(t)$ and quick decrease of C_m reflect the enhancement of the mass transfer process. Results indicate that smaller K and ω lead to larger amplitude of $K(t)$ and quicker reduction of C_m. Moreover, the increase in *De* also can magnify the amplitude of $K(t)$ and decrease the mean concentration C_m. In addition, we find there exists a critical oscillating Reynold number *Re* by analyzing the variations of $K(t)$ and C_m with *Re*. Finally, comparing to the case of Newtonian fluid by setting $De = 0$, a more effective dispersion process for Maxwell fluid can be observed interestingly. The present study is likely to have important bearing on the problem of dispersion of tracers in blood flow through arteries.

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1. Introduction

Microfluidics is one of the most important research areas in MEMS due to its considerable applications in biology, chemistry, medical engineering, drug delivery and so on [1–3]. The fluid flow in MEMS [4] is effectively actuated by employing pressure gradient, magnetic field, electrical field, or their suitable combinations, etc. When solid surfaces contact with electrolytic solutions, it will cause charge exchange at the solid-liquid interface so as to form the electrical double layer (EDL) [5]. Under the influence of an applied electric field along the charged surface tangentially, the excess cations migrate toward the cathode and the excess anions migrate toward the anode in the electrolyte solutions. The migration of the mobile ions will carry surrounding liquid element by viscosity, resulting in an electroosmotic flow (EOF), which has been widely investigated by many researchers [6–11]. Among the study of EOF, electroosmotic force is a crucial actuation mechanism for controlling fluid flow in the microfluidic devices.

All the aforementioned research works are confined to Newtonian fluid. However, there are many fluids with nonlinear relationship between stress and strain in industrial applications. A widespread interest has been developing to envisage flow behavior of non-Newtonian fluids [12], such as crude oil, toothpaste, paint, shampoo and blood, during the past few decades. Some constitutive models, including Jeffery fluid [13], Maxwell fluid [14–18], power law fluid [19,20], third grade fluid [21,22], micropolar fluid [23,24] and casson fluid [25–27], are utilized to capture the essential physics of interest. Liu et al. analytically studied time-periodic EOF of generalized Maxwell fluids through both micro-parallel plates [15] and circular microtubes [16], respectively. They obtained the analytical velocity of the EOF and discussed the effects of the relevant parameters on the flow characteristics in detail.

The mass transfer processes have also received wide attentions for many applications ranging from mixing and transportation of drugs or toxins in physiological systems to chromatographic separations in chemical engineering [28]. For instance, the rate of dispersion as well as concentration distribution plays a crucial role on drug delivery in a cardiovascular system [29]. Many investigations have been carried out concerning different aspects of these dispersion problems. Aris investigated the theory of Taylor dispersion for the steady [30] and pulsatile [31] flow and determined effective diffusion coefficient by adopting the method of momentum moments. Winter et al. [32] studied diffusion in a random velocity field using a formal perturbation expansion. Gill and

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Sankarasubramanian analytically studied the dispersion coefficient as well as concentration distribution with a uniform [33] or non-uniform slug [34] using series expansion method. Many researchers analyzed the solute dispersion in laminar flow with different geometrical shapes, such as parallel-plate [35,36], nearly rectangular cross-sectional shapes [37], microtube [38,39], annulus [40], eccentric annulus [41] and sphere [42]. Hazra et al. [43] presented the dispersion of a viscoelastic fluid through two parallel plate channel in the presence of a periodic pressure gradient. Further they examined the influences of amplitude and frequency of the oscillating pressure and viscoelastic parameter on longitudinal dispersion coefficient and average concentration. Nagarani and Sebastian [44], Manopoulos and Tsangaris [45] also presented solute dispersion drived by periodic pressure gradient in a Casson and Jeffrey fluid, respectively. Ng [46], Rana and Murthy [47] presented the dispersion of chemical species considering the chemical reactions at the tube wall for Newtonian and non-Newtonian fluid, where the pressure gradient consists of steady and oscillatory components.

Recently, the analysis of dispersion phenomena in EOF has attracted attention of many researchers owing to its relevance in fluid transport, mixing and separation. Compared with pressure-driven flow, due to the impacts of imposed electric field, EOF can effectively control and actuate the fluid flow with no moving parts and maintain high reliability and simplicity especially for microfluidic devices. Ng and Zhou [48] addressed the effects of hydrodynamic slip, pressure gradient and slowly varying wall electrical potential on dispersion. Paul and Ng [49] determined the effect of aspect ratio of a rectangular microchannel on the EOF dispersion coefficient.

An area of particularly intensive research has been the use of time-periodic EOF, which has been shown to enhance mixing prevailing in microdevices. Huang and Lai [50] analytically investigated the enhanced mass transfer in an oscillatory EOF in a parallel-plate microchannel. Their results indicated Womersley number, Debye length and tidal displacement have significant influences on mass transport rate. Bhattacharyya and Nayak [51] and Ramon et al. [52] discussed the mass transfer in time-periodic EOF through rectangular and cylindrical tube, respectively. Paul and Ng [53] performed the EOF dispersion excited by AC electrical field through circular channel with oscillating wall potential and analyzed the effects of Debye length, phases of the oscillating fields and wall potential on dispersion.

Based on the previous analysis, however, no one seems to have discussed, to the authors' knowledge, the AC EOF dispersion of non-Newtonian fluid in a microtube. So the purpose of the present study is to provide exact analytic expressions for the longitudinal dispersion coefficient and the mean concentration using series expansion method. By numerical computation, the influences of several nondimensional parameters on the dispersion coefficient and mean concentration are interpreted in detail. The present study is likely to have potential application on the dispersion of tracers in blood flow through arteries.

2. Mathematical formulation

Under an Alternating Current (AC) electrical field, periodic EOF of Maxwell fluid through a microtube is investigated in [16]. Based on this EOF velocity, the dispersion phenomenon of the injected solute is conducted by the following sections in detail. In this paper, the injected solute of is introduced at initial time within a range of band wide x_s^* and its concentration is supposed as C_0^*. The radius of microtube is a and its pipe length is L. It is assumed that the pipe length L is much greater than the pipe radius a, i.e., $L \gg 2a$. Wall electrical potential ψ_0^* is assumed to be uniform throughout the surface of the microtube and its value is always negative. The physical model is sketched in Fig. 1 and the cylindrical coordinate system is established at the central part of the microtube where x^*-axis is oriented along the flow direction and r^*-axis is perpendicular to the charged surface. An AC electrical field $E_x^* = E_0 \sin(\omega^* t^*)$ is externally imposed along x^*-axis direction where E_0 is the amplitude of the AC electric field, ω^* is the oscillating angular frequency and t^* is the time. The shadowed section in Fig. 1 represents the solute distribution at initial injection. The superscript '$*$' represents the dimensional parameter.

2.1. Electrical potential distributions

For an axial symmetric electrolyte solution in the tube, the electrical potential $\psi^*(r^*)$ and the local volumetric net charge density $\rho_e^*(r^*)$ are described by the following Poisson-Boltzmann (PB) equations [54]

$$\frac{d^2\psi^*}{dr^{*2}} + \frac{1}{r^*}\frac{d\psi^*}{dr^*} = -\frac{\rho_e^*(r^*)}{\varepsilon}, \tag{1}$$

$$\rho_e^*(r^*) = (n^+ - n^-)ze = -2n_0 ze \sinh\left[\frac{ze\psi^*}{k_b T}\right], \tag{2}$$

where ε is the dielectric constant of the electrolyte liquid, n^+ and n^- is the positive and negative ions number concentration respectively, z is the valence of ions, e is the electron charge, n_0 is the ionic number concentration of bulk liquid, k_b is the Boltzmann constant and T is the absolute temperature.

If the electrical potential of the EDL is small compared with the thermal energy of the charged species, i.e., $(|ze\psi^*|<|k_bT|)$, the hyperbolic sine function in the right hand side of Eq. (2) can be approximated by the $ze\psi^*/k_bT$ (the Debye-Hückel approximation). The solution to PB Eq. (1) can be written as $\psi^*(r^*) = \psi_0{}^* I_0(\kappa r^*)/I_0(\kappa a)$ under the boundary conditions $\psi^*|_{r^* = a} = 0$ and $d\psi^*/dr^*|_{r^* = 0} = 0$. Here I_0 is modified Bessel function of first kind of order zero. Further, the net charge density can be written as

$$\rho_e^*(r^*) = -\varepsilon\kappa^2\psi_0^* I_0(\kappa r^*)/I_0(\kappa a). \tag{3}$$

where κ is the Debye-Hückel parameter, $\kappa = (2n_0 z^2 e^2/\varepsilon k_b T)^{1/2}$, $1/\kappa$ refers to the characteristic thickness of the EDL, ψ_0^* is the wall zeta potential.

2.2. Analytical solutions of velocity field

Due to the symmetry of geometry and the narrow-long feature of microtube, the axial velocity is independent on θ^* and x^*. The fluid velocity only has a component in the x^* direction driven by imposed AC electric field force. So the axial velocity can be written as $u^* = u^*(r^*, t^*)$. The motion equation and the constitutive equation for the EOF is given as [15]

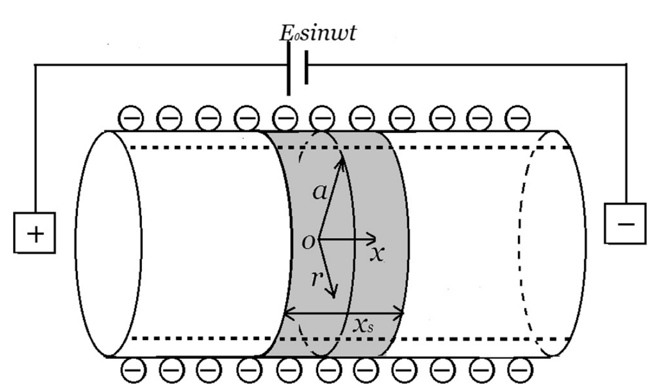

Fig. 1. Schematic of the physical model and the concentration distribution at $t = 0$.

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