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Dimensionless pressure drop number for non-newtonian fluids applied to Constructal Design of heat exchangers

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ABSTRACT

This paper introduces a dimensionless group for pressure drop, named Bejan number (*Be*), to be used with non-Newtonian fluids. When defining *Be* for non-Newtonian fluids, it is necessary to choose a characteristic apparent viscosity to compose this dimensionless group. In non-Newtonian fluid dynamics, the viscosity at a characteristic shear rate is usually chosen as reference, with the latter given as the reference velocity divided by the reference length. When the flow rate is not known, a reference velocity may be taken as the square root of the pressure drop divided by the mass density. Thus, a characteristic apparent viscosity may be defined for any non-Newtonian model, even for one that does not present a characteristic viscosity defined explicitly in the viscosity function, such as the power-law model. The non-dimensionalization of motion equations for the crossflow of a power-law fluid between two aligned cylinders was performed using this philosophy. Some numerical tests were performed to corroborate the idea that the introduced form for *Be* is a good alternative to be used in experiments to predict and evaluate the heat transfer density in the context of Constructal Design of heat exchangers tube bundles.

1. Introduction

Constructal Design is a method to assess the effect of geometric parameters on the performance of systems with the objective of providing easier access to the currents that flow through them [7,8,6]. This method has been employed elsewhere to investigate the design of tube bundles to maximize the heat transfer density [24,17,10,18]. Due to designs that increase heat transfer by convection also increase friction, a good choice in this kind of investigation is to look for optimal geometries for fixed values of pressure drop. This has been the method employed in works guided by Constructal Design Method, such as [27,24,10,14]. Constructal Design has also been applied to discover best configurations by important contributions in other domains dealing with transport phenomena, e.g. the transport of ionic species through a porous medium by means of electrokinetics [16] and minimize the diffusion transfer resistance and determine the macroscopic diffusion coefficient [29].

In order to investigate such problems using dimensionless parameters, a dimensionless pressure drop parameter is needed, to be used as a constraint in a constant pressure drop analysis. A pioneer paper by Battacharjee and Grosshandler [11] has introduced a dimensionless pressure drop parameter to the analysis of a jet and suggested the name Bejan number for this parameter. In their work, the Bejan number was defined as

$$Be = \frac{\Delta p L^2}{\mu \nu}, \tag{1}$$

since the problem did not involve heat transfer. Later, Petrescu [20] observed the similarity between this dimensionless group and the dimensionless pressure drop parameter introduced in the paper of Bejan and Sciubba [9] about forced convection between parallel plates. They named this parameter Bejan number. This Bejan number had the form

$$Be = \frac{\Delta p L^2}{\alpha \mu}. \tag{2}$$

Stanescu et al. [27] employed a pressure drop based Reynolds number, and Rocha and Bejan [24] also employed dimensionless pressure drop parameter referred to as pressure drop number. In the paper by Bello-Ochende and Bejan [10], the name Bejan number was finally adopted to designate the pressure drop number, and the balance equations were non-dimensionalized using only

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Nomenclature

Be	Bejan number, $Be = \Delta p L^2/\alpha\mu$
Be_{NN}	non-newtonian Bejan number, $Be_{NN} = \Delta p L^2/\alpha\eta_c$
c_p	specific heat
D	diameter
D_{ij}	strain rate tensor
$\tilde{D}_{ij}$	dimensionless strain rate tensor
f	Fanning friction factor
K	consistency index
k	thermal conductivity
L	characteristic length
L_d	downstream flow length
L_u	upstream flow length
n	flow index
p	pressure
$\tilde{p}$	dimensionless pressure
Pr	Prandtl number, $Pr = \mu c_p/k$
Pr_{NN}	non-newtonian Prandtl number, $Pr_{NN} = \eta_c/\rho\alpha$
q'	heat transfer rate per unit length
$\tilde{q}$	dimensionless heat transfer density
Re	Reynolds number
S_0	spacing between cylinders
T	temperature
T_0	fluid temperature
T_w	wall temperature
$\tilde{T}$	dimensionless temperature
U	characteristic velocity
u_{ave}	average velocity
u_i	velocity vector
$\tilde{u}_{ave}$	dimensionless average velocity
$\tilde{u}_i$	dimensionless velocity vector
x_i	position vector
$\tilde{x}_i$	dimensionless position vector
α	thermal diffusivity
δ	general diffusivity
Δp	pressure drop
η	viscosity function
η_c	characteristic viscosity
η_0	zero shear rate viscosity
η_∞	infinite shear rate viscosity
η_p	plastic viscosity
$\tilde{\eta}$	dimensionless viscosity function
λ	time coefficient
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density
$\dot{\gamma}_c$	characteristic shear rate
τ	stress magnitude
τ_0	yield stress
τ_{ij}	extra-stress tensor
$\tilde{\tau}_{ij}$	dimensionless extra-stress tensor

the Bejan and Prandtl numbers as similarity parameters. Joucaviel et al. [14] also employed this formulation in the Constructal Design of rotating cylinders in cross-flow.

Awad [2] introduced a modified Bejan number for mass transfer applications, and Awad and Lage [3] introduced a general Bejan number, in which the diffusivity, δ, of the kind of process under consideration is employed in the denominator in the form

$$Be = \frac{\Delta p L^2}{\rho\delta^2}. \tag{3}$$

This formulation avoids that the Prandtl number be in the momentum equation in the resulting non-dimensionalized governing equations as in Bello-Ochende and Bejan [10] and Joucaviel et al. [14].

Regarding Constructal Design for heat transfer in tube bundles, in order to use a dimensionless pressure drop to non-Newtonian fluids, a characteristic viscosity, η_c, must be defined. The usual scaling method for purely viscous non-Newtonian fluids comprises using η_c as the apparent viscosity at a characteristic shear rate, $\dot{\gamma}_c$. As $\dot{\gamma}_c$, it is usual to employ the rate characteristic velocity-characteristic length, U/L e.g. [5,13,22,19]. Some authors, as Kozicki et al. [15], Rao [21] and Chabra and Richardson [12], assume that the characteristic viscosity should be taken as the value which keeps the relation $f = 16/Re$ true for laminar fully developed flow through a channel of arbitrary but uniform cross-section. This choice is controversial when dealing with external flows, and it is also dependent on having an analytical or even experimental result for the internal flow. The choice of a characteristic viscosity or a characteristic shear rate has been a subject of discussion in papers such as Souza Mendes [26] and Thompson and Soares [28]. These authors argue that η_c should be a rheological parameter of the fluid, such as the infinite-shear-rate viscosity. Other works employ η_c as the viscosity calculated at an exact place in the problem domain. However, this option is weak because η_c is only known a posteriori, when the flow is already solved. The shortcoming in using a rheological η_c is that, when using a fluid model such as power-law, a rheological η_c is not defined.

In this work, a form to obtain a characteristic viscosity for a power-law fluid based on an imposed pressure drop is introduced. This formulation may be employed to the Constructal Design of heat exchangers, following the methodology presented in Bello-Ochende and Bejan [10] and Joucaviel et al. [14].

2. Background

The basic idea for the introduction of a dimensionless number based on pressure drop is to scale the velocity components using not a reference velocity, as usual, but a reference pressure drop. Bejan [9] has introduced, in the context of Newtonian fluids, the following scaling:

$$\tilde{x}_i = \frac{x_i}{L}; \ \tilde{u}_i = \frac{u}{\Delta p L/\mu}; \ \tilde{p} = \frac{p}{\Delta p}; \ \tilde{T} = \frac{T - T_0}{T_w - T_0}, \tag{4}$$

where x_i is the position vector, u_i is the velocity vector, p is the pressure, L is a characteristic length, Δp is the reference pressure drop and μ is the fluid viscosity. The dimensionless temperature is also employed in terms of two reference temperatures T_0 and T_w. Note that a characteristic velocity is given by the relation $\Delta p L/\mu$, which relates the flow rate to the pressure drop and fluid viscosity, as one would probably expect.

In this case, the usual equations for incompressible flow, namely the continuity equation, the momentum equations and the energy balance equation in terms of temperature may be written on their dimensionless form, respectively as:

$$\frac{\partial \tilde{u}_i}{\partial \tilde{x}_i} = 0, \tag{5}$$

$$\frac{Be}{Pr}\tilde{u}_j\frac{\partial \tilde{u}_i}{\partial \tilde{x}_j} = -\frac{\partial \tilde{p}}{\partial \tilde{x}_i} + \frac{\partial^2 \tilde{u}_i}{\partial \tilde{x}_i \partial \tilde{x}_i}, \tag{6}$$