International Journal of Heat and Mass Transfer 115 (2017) 1055–1063

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Numerical modeling of frost growth and densification on a cold plate using frost formation resistance

Chiwon Kim, Jaehwan Lee, Kwan-Soo Lee *

School of Mechanical Engineering, Hanyang University, 222 Wangsimni-ro, Seongdong-gu, Seoul 04763, Republic of Korea

ARTICLE INFO

Article history:
Received 1 March 2017
Received in revised form 31 July 2017
Accepted 31 July 2017
Available online 7 August 2017

Keywords:
Frost thickness
Frost density
Numerical analysis
Cold plate

ABSTRACT

Frost formation on a cold plate was modeled numerically using the frost formation resistance. A multiphase Eulerian method was used to model the humid-air and frost phases. The grids in the computational domain were classified into three zones, and a frost density threshold was suggested to construct an algorithm of frost growth and densification. To verify the present model, analyses were performed under various operating conditions, and the results were compared with published experimental results. In addition, the frost formation resistance was verified by analyzing the frost formation resistance profiles on the frost surface and at the frost inside, and temperature of the frost and absolute humidity of the humid-air inside the frost were also predicted using the verified model. This showed that the frost formation resistance matched the general well-known behavior of frost formation. The temperature of the frost and absolute humidity of the humid-air in the frost showed larger variation near the frost surface, which has lower frost density, than near the cold plate, which has higher frost density.

1. Introduction

Frost formation is a phase change phenomenon that occurs when the water vapor in humid air meets a cold surface. The formed frost reduces the heat transfer between the air and the cold surface, but also physically blocks the flow. For this reason, frost formation is considered a major issue [1] in various engineering problems related to heat pumps and aerospace engineering From a methodological view, experimental research methods require much time and cost, and there are restrictions on the setting of operating conditions. Therefore, it is essential to develop a numerical analysis model that predicts frost formation and can be applied to various shapes and operating conditions.

Existing frost formation models can be classified as analytic solution models and numerical analysis models. Among the analytic solution models, Tao et al. [2,3] proposed a one-dimensional analytic model for frost only. Lee et al. [4] modeled the frost growth and densification including frost inside mathematically, and presented a humid-air-frost coupled model. Yang et al. [5–7] developed a frost formation model under turbulent conditions and applied it to a heat exchanger. Na and Webb [8,9] introduced supersaturation conditions on the frost surface to enhance the accuracy of the one-dimensional model. Kandula [10,11] improved the accuracy of the one-dimensional model by introducing a correlation of frost density and frost thermal conductivity, which was applicable to a wide range of operating conditions. However, these studies based on a one-dimensional analytic solution have limited ability to extend to various shapes or to predict the variation of physical quantities in the frost.

The numerical analysis models can be subdivided into humidity gradient mass transfer models and supersaturation mass transfer models. Most relevant studies use a humidity gradient mass transfer, which is similar to the mass transfer of the analytic solution models. Lenic et al. [12] expressed the mass transfer using the frost density and an absolute humidity gradient. Kim et al. [13] suggested a modified Sauter mean diameter to directly introduce the mass transfer of the existing analytic solution models into the numerical analysis model. Armengol et al. [14] improved the model accuracy by correcting the material diffusion coefficients and frost growth algorithms. However, these methods can be considered extensions of the analytic solution models that consider only the mass transfer through the frost surface. Hence, they are fundamentally incompatible with the numerical analysis model, which requires consideration of both frost surface and frost inside. The supersaturation mass transfer model is more suitable for numerical analysis but only recently appeared; thus, there are relatively few reports on this model. Wu et al. [15,16] defined the volumetric mass transfer rate using supersaturation and the velocity of humid air, and they improved the modeling accuracy by dividing

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* Corresponding author.
E-mail address: ksleehy@hanyang.ac.kr (K.-S. Lee).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.07.130
0017-9310/

Nomenclature

A	area [m^2]
a	correction constant
d	distance from cold plate [m]
h_{sub}	sublimation enthalpy of water [J/kg]
k	thermal conductivity [W/(m K)]
L	cold-plate length [m]
$\dot{m}'''$	volumetric mass transfer rate [kg/m^3 s]
$\dot{Q}''$	heat flux [W/m^2]
R	frost formation resistance [m^3 s/kg]
S	source
T	temperature [K or °C]
T_0	freezing temperature of water [K or °C]
t	time [s]
u	velocity [m/s]
V	volume [m^3]
x	coordinate [m]

Greek symbols

α	volume fraction
δ_f	frost thickness [m]
ε	error [%]
$\Delta\Phi$	frost formation driving potential [kg/kg]
ϕ	scalar
Γ	diffusion coefficient
ρ	density [kg/m^3]
$\rho_{f.th}$	frost density threshold [kg/m^3]
ρ_{ice}	ice density, $\rho_{ice} = 915$ [kg/m^3]
ω	mass fraction or absolute humidity [kg/kg]

Subscripts

cv	control volume or grid
cz	crust zone
f	frost or frost phase
fz	frost zone
h	humid-air phase
i	index, $i = 1, 2, 3, \ldots$
j	index, $j = 1, 2, 3, \ldots$ or $j = x, y$
k	kth time step
m	phase, $m = h$ or f
n	species, n = dry air or water vapor
t	total
p	cold plate
sat	saturation

the computational domain into humid air and frost regions in real time. They also improved the model by introducing a phase change driving force [17]. However, this study was conducted under only a few sets of operating conditions.

In this study, the frost growth and densification were modeled based on a numerical analysis. The modeling was performed using source terms composed of frost formation resistance and supersaturation under various operating conditions. The proposed model was verified using existing experimental results. The overall and local characteristics of the frost were predicted and analyzed based on the results of the model.

2. Mathematical modeling

2.1. Problem description

Fig. 1 illustrates the problem definition for frost formation on a cold plate. Fig. 1(a) shows the computational domain for frost formation with $D/L = 2.0$ and $H/L = 0.5$. There are a humid-air phase, which is composed of dry air and water vapor, and a frost phase in the computational domain. The humid air enters from the left inlet, loses some water vapor to the frost, and exits through the right outlet. The water vapor collected by the frost increases the frost thickness and density through phase change from water vapor to frost.

In a two-dimensional computational domain of a finite volume method (FVM), it is impossible to express the exact shape and position of the frost surface using a one-dimensional line because the shape and position changes in real time. Thus, a two-dimensional concept of frost crust is introduced to represent the frost surface. Fig. 1(b) shows an example of grid zone classification in the computational domain. The dotted line indicates the frost surface at a certain time. The air zone (grid A), the crust zone (grid B) and frost zone (grid C) represent the air, frost surface, and frost, respectively. The crust zone is a single layer of grids which contain the actual frost surface. The crust zone forms the outermost boundary of the frost zone and is adjacent to the air zone. The zones are mathematically defined as:

$$\begin{cases} \text{Air zone}: & \rho_f = 0 \\ \text{Crust zone}: & 0 \leqslant \rho_f < \rho_{f.th} \\ \text{Frost zone}: & \rho_{f.th} \leqslant \rho_f \end{cases} \tag{1}$$

where ρ_f and $\rho_{f.th}$ are the frost density and frost density threshold, respectively. The frost density threshold is described in Sections 2.5 and 3.2.

2.2. Governing equations

The following assumptions are introduced for the computational analysis modeling of frost formation on a cold plate.

(1) The air is an ideal gas.
(2) The thermal conductivity of frost is a function of the only frost density.

A multi-phase Eulerian model was used for the two-phase flow modeling with a single computational domain. The two considered phases are a humid-air phase, which is consists of dry air and water vapor species, and a frost phase. The conservation law for an arbitrary physical quantity ϕ in an arbitrary phase m is:

$$\frac{\partial \rho_m \phi_m}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho_m u_{j.m}\phi_m - \alpha_m \Gamma_m \frac{\partial \phi_m}{\partial x_j}\right) = S_m \tag{2}$$

where Γ_m and S_m are the diffusion coefficient and source term, respectively. Table 1 shows the diffusion coefficient and source term for each conservation law. α_m is the volume fraction for phase m. The total sum of all volume fractions in each grid is 1:

$$\alpha_h + \alpha_f = 1 \tag{3}$$

where α_h and α_f are the volume fractions for the humid-air phase and frost phases, respectively. The volume fraction of the frost phase is defined as:

$$\alpha_f = \frac{\rho_f}{\rho_{ice}} \tag{4}$$

where ρ_f and ρ_{ice} are the frost density and ice density, respectively.