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Three-equation local thermal non-equilibrium model for transient heat transfer in porous media: The internal thermal conduction effect in the solid phase

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ABSTRACT

The present work developed a three-equation local thermal non-equilibrium (LTNE) model to model the initial effect on the internal thermal conduction in the solid phase of porous medium. The two energy equations in the original LTNE model were extended with a third equation to govern the dimensionless thermal penetration depth for the internal thermal conduction in the solid phase, *I*, which can describe the initial effect. The model was validated by comparisons between an analytical solution of the *I* equation and exact solutions of thermal conduction problems and between the macro-scale and the pore-scale numerical simulations. The comparisons demonstrate that the three-equation LTNE model is more accurate than the two-equation LTNE model with a constant *I* during the initial period. An important similarity number, η, is obtained for the enhanced geothermal system (EGS) heat extraction problem from the dimensionless three-equation LTNE model, which measures the internal thermal conduction effect in the solid phase of a porous medium. The initial effect in the EGS heat extraction can be neglected for $\eta < 0.5$. Criteria for degeneration of the three-equation LTNE model are proposed.

1. Introduction

Transient heat transfer in porous media is widely used in enhanced geothermal systems (EGS), energy storage, thermal enhanced oil recovery, chemical process engineering, and nuclear energy. The local thermal equilibrium (LTE) model and the local thermal non-equilibrium (LTNE) model have been used to describe these heat transfer processes in the macro scale. The LTE model assumes local thermal equilibrium in the porous media, while the LTNE model includes the local temperature difference between the solid and fluid phases in the porous media. The LTNE model then gives better predictions for situations in which the assumption of local thermal equilibrium is invalid. Many transient heat transfer problems in porous media have been modeled using the LTNE model [1–11]. The LTNE model can be expressed as

$$\begin{cases} \varepsilon\rho_f c_{pf}\frac{\partial\langle T_f\rangle^f}{\partial t}+\rho_f c_{pf}\langle\mathbf{u}\rangle\nabla\langle T_f\rangle^f=\nabla(k_{f,eff}\nabla\langle T_f\rangle^f)+ah_{sf}(\langle T_s\rangle^s-\langle T_f\rangle^f) \\ (1-\varepsilon)\rho_s c_{ps}\frac{\partial\langle T_s\rangle^s}{\partial t}=\nabla(k_{s,eff}\nabla\langle T_s\rangle^s)-ah_{sf}(\langle T_s\rangle^s-\langle T_f\rangle^f) \end{cases} \tag{1}$$

where h_{sf} is the overall solid-to-fluid heat transfer coefficient and a is the interfacial area per unit volume.

The absent of local thermal equilibrium usually results from a relatively small volumetric heat transfer coefficient, ah_{sf}. Thus, the accuracy of h_{sf} has an important effect on the LTNE model predictions. Many experimental approaches have been used to determine the solid-surface-to-fluid heat transfer coefficient, $h_{sf,w}$ [12,13]. If the Biot number based on the size of the a solid matrix element, $\mathrm{Bi} = h_{sf,w}d_p/k_s$, is much less than one, the internal solid thermal resistance in porous media can be neglected [14]. In that case, h_{sf} is approximately equal to $h_{sf,w}$, with $h_{sf,w}$ commonly used in porous media heat transfer models [15–17]. If not, the thermal conduction in the solid material in the porous media must be considered. For example, the reservoirs of the EGS consist of fractured rocks where the size of each piece of rock is much larger than that of the fracture. Thus, the reservoirs can be regarded as porous media with a very large Biot number where the thermal resistance inside the rocks is very important [18]. The solid thermal conduction cannot be ignored in packed beds used for thermal energy storage [19] and chemical engineering [20] neither. Kaviany [21] and Quintard et al. [22] discussed the influence of the internal solid thermal conduction on h_{sf}.

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Nomenclature

a	interfacial area per unit volume
Bi	Biot number of a solid matrix element
c_p	specific heat
d_p	sizes of the solid matrix elements (slab thickness or cylinder or sphere diameter)
D	dimensionless principle number defined by Eq. (52)
Fo	Fourier number of a solid matrix element
G	mass flow rate
h_{sf}	overall solid-to-fluid heat transfer coefficient
$h_{sf,w}$	solid-surface-to-fluid heat transfer coefficient
H	height
I	dimensionless thermal penetration depth for internal thermal conduction in the solid phase, defined by Eq. (6)
k	thermal conductivity
L	length
m	shape factor ($m = 1$ for large slabs, 2 for long cylinders, and 3 for spheres)
N	number of fractures
$\dot{q}_s$	volumetric heat source in the solid phase
q_{sf}	solid-to-fluid interfacial heat flux
r	radial coordinate in each solid matrix element
R	dimensionless radial coordinate
t	time
T	temperature
$\langle T_d \rangle_e^s$	penetration temperature difference
$\mathbf{u}$	fluid velocity vector
u	fluid velocity component at the x coordinate direction
U	dimensionless velocity defined by Eq. (55)
W	width
$\mathbf{x}$	position vector
x	x coordinate
X	dimensionless x coordinate
y	y coordinate

Greek symbols

β	fluid-to-solid effective volumetric heat capacity ratio
ε	porosity
γ	closure coefficient for the solid thermal conductance, defined by Eq. (20)
μ	dynamic viscosity
ξ	dimensionless x coordinate
ρ	density
τ	dimensionless time
η	similar principle number defined by Eq. (60)

Subscripts

0	initial
a	advection
c	conduction
const	constant
eff	effective
f	fluid phase
in	inlet
s	solid phase
ss	steady state
qss	quasi-steady state
w	solid-to-fluid interface
LTE	LTE model
MEGS	mini enhanced geothermal system
LEGS	large enhanced geothermal system

Other

$\langle\ \rangle$	phase average over a representative elementary volume
$\langle\ \rangle^f$	intrinsic average for the fluid phase over a representative elementary volume
$\langle\ \rangle^s$	intrinsic average for the solid phase over a representative elementary volume
$\langle\ \rangle_e^s$	intrinsic average for the solid phase over a solid phase unit cell
$\langle\ \rangle^w$	area average on the solid-to-fluid interfacial surface

There are two main approaches to include the internal thermal conduction in porous media. The first approach includes a constant solid thermal resistance in the overall solid-to-fluid heat transfer coefficient. Stuke [23] proposed the following model:

$$h_{sf}^{-1} = h_{sf,w}^{-1} + \left(\frac{k_s}{Id_p}\right)^{-1} \tag{2}$$

where $I \equiv 1/6$ for large slabs, 1/8 for long cylinders, and 1/10 for spheres. Dixon and Cresswell [24] then used this model to analyze the heat transfer in packed beds with the LTNE model. Grangeot et al. [25] and Rees [26] proposed similar constant solid thermal resistance models for other solid shapes. This approach of assuming a constant solid thermal resistance is simple but loses some dynamic information for the transient thermal conduction in the solid phase.

The second approach is to use the so-called dispersion-particle-based model, first developed by Handley and Heggs [27]. This model couples a radial thermal conduction equation for the particles with the macro-scale fluid temperature equation. This model only requires the parameter of $h_{sf,w}$ instead of h_{sf} for the LTNE model and can be written as [21]:

$$\begin{cases} \varepsilon\rho_f c_{pf}\frac{\partial\langle T_f\rangle^f}{\partial t} + \rho_f c_{pf}\langle\mathbf{u}\rangle\nabla\langle T_f\rangle^f = \nabla(k_{eff}\nabla\langle T_f\rangle^f) + ah_{sf,w}(T_s|_w - \langle T_f\rangle^f) \\ \rho_s c_{ps}\frac{\partial T_s}{\partial t} = \frac{k_s}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial T_s}{\partial r}) \\ -k_s\frac{\partial T_s}{\partial r}\big|_w = h_{sf,w}(T_s|_w - \langle T_f\rangle^f) \end{cases} \tag{3}$$

This model more accurately includes dynamic information about the transient thermal conduction inside the solid particles as shown by Ismail and Stuginsky [28] and Saez and McCoy [14]. However, this model has two disadvantages compared to the LTNE model. First, this model adds one dimension (the pore-scale r coordinate) for the calculation. Second, the prediction of the macro-scale thermal conduction is not as accurate as that of the LTNE model due to neglect of macro-scale temperature gradient in the solid phase.

Therefore, an LTNE model is modeled which considers the dynamic information about the transient thermal conduction in the solid phase. Preliminary investigations have been conducted on this topic with assuming that $h_{sf,w}$ was large enough. Grangeot et al. [25] found that the time-dependent $h_{sf}(t)$ decreases to a constant from a very large value after a sudden change in the fluid temperature. This implies that $h_{sf}(t)$ has an initial effect for a step response. However, they argued that the initial effect can be neglected for their cases and did not discuss it any further. They also pointed out that the classical LTNE model cannot model high-frequency temperature oscillations. To address on this problem, Auriault and Royer [29] proposed adding a time derivative of the macro-scale temperature in the internal heat transfer term for pure conduction in a double diffusive porous medium.

Thus, the LTNE model cannot accurately predict the initial effect on the internal transient thermal conduction in the solid phase. Ouyang et al. [30] have tried to combine these two approaches