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Constructal design for helm-shaped fin with internal heat sources

Huijun Feng, Lingen Chen *, Zhihui Xie, Fengrui Sun

Institute of Thermal Science and Power Engineering, Naval University of Engineering, Wuhan 430033, PR China
Military Key Laboratory for Naval Ship Power Engineering, Naval University of Engineering, Wuhan 430033, PR China
College of Power Engineering, Naval University of Engineering, Wuhan 430033, PR China

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ABSTRACT

A helm-shaped fin (HSF) with single and multiple internal heat sources (IHSs) are investigated. Based on constructal theory, the volumes of the HSF and IHS are set as the constraints. The dimensionless maximum thermal resistance (DMTR) of the model is selected as the optimization objective, and its optimal construct is obtained. The results show that for the HSF with single IHS, the dimensionless thickness $\tilde{W} = 0.33$ should be avoided in the constructal design of the model to improve its heat transfer performance (HTP). For the HSF with multiple IHSs, the DMTR has its double minimum, and it can be further decreased by increasing the IHS's number n and convective heat transfer parameter a, respectively. The DMTR of the optimal construct with $n = 4$ is decreased by 15.35% than that with $n = 2$. The DMTR of the model with HSF and nonuniform heat generation is decreased by 6.57% compared with that with circular-shaped fin. Moreover, the optimal constructs of the models with uniform and nonuniform heat generations are different, and a more uniform heat generation of the IHS leads to a better HTP of the model with HSF. The results obtained in this paper can provide some theoretical guidelines for the fin designs of real thermal systems.

1. Introduction

Due to the extended surfaces, heat can be effectively removed from the thermal systems with the help of the fins. Suitable structures of the fins will help to enhance heat transfer performance (HTP); on the contrary, unsuitable structures will restrain HTP. Therefore, many scholars carried out deep studies on the structure optimizations of various fins, such as rectangular fins [1–4], T-shaped fins [5–10], Y-shaped fins [11–13], T-Y-shaped fins [14–16], tree-shaped fins [17–19], leaf-shaped fins [20,21], cylindrical fins [22–27], annular fins [28–31] and heat exchanger fins [32–35].

There are many approaches to carry out heat transfer enhancements [36–40], such as constructal theory [41–60], entropy generation minimization [61–65] and entransy theory [66–71]. For a heat transfer system, constructal theory focuses on its structure optimization, and the theories of entropy generation and entransy focus on the reductions of heat transfer irreversibilities. Among these studies above, fin structure optimization based on constructal theory is one of the hot issues. Bejan and Almogbel [5] investigated the T-shaped fin (TSF) based on constructal theory and analytical method, and increased the thermal conductance of the TSF by 29% than that of the reference fin. Lorenzini and Moretti [6] performed the study of the TSF based on CFD method, and validated the validity of the analytical solution in Ref. [5]. Based on the TSF model with constant convective heat transfer (CHT) coefficient and thermal conductivity, Bhanja and Kundu [7,8], Lorenzini et al. [9] and Hazarika et al. [10] further considered the fin models with radiation effects [7], variable thermal conductivity [8], variable CHT coefficient [9] and combined heat and mass transfer [10], respectively, and offered practical guidelines for the TSF designs. To further improve the HTPs of the TSFs, their structures became more complex, and Y- [11–13], T-Y- [14–16], tree- [17–19] and leaf- [20,21] shaped fins had been proposed.

Besides the TSF, cylindrical fin is another hot research object of the constructal design. Almogbel and Bejan [22] built the pin fin models with cylindrical and cone-shaped branches, respectively, and increased thermal conductances of the fin models by optimizing their external and internal shapes, respectively. Bejan and Almogbel [5] investigated an umbrella cylindrical fin based on constructal theory, and obtained a maximum heat transfer rate by optimizing its length and diameter ratios, simultaneously. Bello-Ochende et al. [23,24] optimized the structures of four cylindrical fins based on heat transfer rate maximization, and proved that the HTPs of the fin models could be improved by adopting nonuniform diameters and heights of the fins. Hajmohammadi et al.

* Corresponding author at: Institute of Thermal Science and Power Engineering, Naval University of Engineering, Wuhan 430033, PR China.
E-mail addresses: lgchenna@yahoo.com, lingenchen@hotmail.com (L. Chen).

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Nomenclature

a	convective heat transfer parameter
b	constant
e	distance, m
h	heat transfer coefficient, W/m^2/K
k	thermal conductivity, W/m/K
N	number of sectorial extended bodies
n	number of the internal heat sources
q'''	volumetric heat generation rate, W/m^3
R	radius, m
R_t	maximum thermal resistance, K/W
T	temperature, K
V	volume, m^3
W	height, m
x	horizontal direction, m
y	longitudinal direction, m
z	third dimensional direction, m

Greek symbols

α	angle, rad
ϕ	volume fraction

Subscripts

h	heat source
f	fin
m	minimum
max	maximum
mm	double minimum
opt	optimal
oo	twice optimal
Var	variable volumetric heat generation

Superscript

$\sim$	dimensionless

[25,26] built the square- and circular-shaped fins with internal heat sources (IHSs), respectively, and obtained the temperature distributions of the two-dimensional models based on finite element method. The results showed that the HTPs of the fin models were effectively improved when the optimal IHS distributions were adopted. Based on the model in Ref. [26], Gong et al. [27] further built a three-dimensional cylindrical fin model with IHSs, and proved that the height of the fin had evident influence on the HTP of the fin model.

Based on the two-dimensional circular-shaped fin model in Ref. [26] and three-dimensional cylindrical fin model in Ref. [27], a three-dimensional helm-shaped fin (HSF) with IHSs will be considered in this paper. Its structure optimization will be implemented by using constructal theory, and the dimensionless maximum thermal resistance (DMTR) will be obtained. The effects of some fin parameters on its HTP will be analyzed.

2. Constructal design for HSF with single IHS

A HSF with an IHS is shown in Fig. 1. The fin is made by one cylinder and a number (N) of sectorial extended bodies (SEBs).

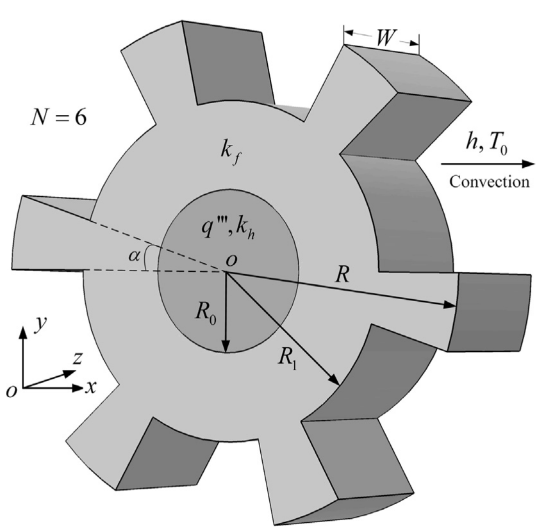

Fig. 1. HSF with single IHS.

The radiuses of the IHS, cylinder and SEB are R_0, R_1 and R, respectively, and the angle of the SEB is α. The height of the model is W, and the volumetric heat generation rate of the IHS is q'''. The heat generated in the heat source first enters the HSF, and then dissipates from the fin surface into the environment with the help of CHT. The materials of the heat source and fin are isotropic, and the thermal conductivities of the two materials are k_h and k_f ($k_f > k_h$), respectively. The surface CHT coefficient (h) is viewed as a constant, and the corresponding environment temperature of the CHT is T_∞.

The heat conduction equations in the fin and heat source are, respectively, given as

$$k_f\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)=0 \tag{1}$$

$$k_h\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+q'''=0 \tag{2}$$

where k_f and k_h are the thermal conductivities of the fin and heat source, and q''' is the volumetric heat generation rate.

The dimensionless temperature and geometrical parameters can be, respectively, written by

$$\tilde{T}=\frac{T-T_\infty}{q'''V^{2/3}/k_f} \tag{3}$$

$$(\tilde{x},\tilde{y},\tilde{R},\tilde{R}_1,\tilde{R}_0,\tilde{W})=\frac{(x,y,R,R_1,R_0,W)}{V^{1/3}} \tag{4}$$

where the volume of the model is $V=\pi R^2 W$.

When the volumes of the model, IHS and HSF are specified, the dimensionless equations of the three constraints can be given as

$$1=\pi\tilde{R}^2\tilde{W} \tag{5}$$

$$\phi_0=\pi\tilde{R}_0^2/(\pi\tilde{R}^2) \tag{6}$$

$$\phi_1=\left[\pi(\tilde{R}^2-\tilde{R}_1^2)\cdot N\alpha/(2\pi)+\pi\tilde{R}_1^2\right]/(\pi\tilde{R}^2)-\phi_0 \tag{7}$$

where the volume fractions $\phi_0(=V_0/V)$ and $\phi_1(=V_f/V)$ are the ratios of the IHS and HSF volumes to the model volume, respectively.