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An alternative unsteady adaptive stochastic finite elements formulation based on interpolation at constant phase

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ABSTRACT

The unsteady adaptive stochastic finite elements method based on time-independent parametrization (UASFE-ti) is an efficient approach for resolving the effect of random parameters in unsteady simulations. It achieves a constant accuracy in time with a constant number of samples, in contrast with the usually fast increasing number of samples required by other methods. In this paper, an alternative unsteady adaptive stochastic finite elements formulation based on interpolation at constant phase (UASFE-cp) is developed to further improve the accuracy and extend the applicability of UASFE-ti. In addition to achieving a constant number of samples in time, interpolation at constant phase: (1) eliminates the parametrization error of the time-independent parametrization; (2) resolves time-dependent functionals, which cannot be modeled by the parametrization; and (3) captures transient behavior of the samples, which is an important special case of time-dependent functionals. These three points are illustrated by the application of UASFE-cp to random parameters in a mass–spring–damper system, the damped nonlinear Duffing oscillator, and an elastically mounted airfoil with nonlinearity in the flow and the structure. Results for different types of probability distributions are compared to those of UASFE-ti and Monte Carlo simulations.

1. Introduction

It is recognized in the engineering community that there is an increasing need to move towards unsteady simulations in computational fluid dynamics. For example, large model inaccuracies in steady RANS simulations of turbulent flow give rise to more accurate unsteady turbulence modeling. Continued grid refinement in steady computations can also result in unsteady behavior, such that a steady grid converged solution might not be as well-defined as generally assumed. This trend towards unsteady simulations will also dictate an increasing application of uncertainty quantification methods to unsteady problems.

The most widely used methods for uncertainty quantification include Monte Carlo simulation [9], (non-intrusive) polynomial chaos methods [1,8,10,20,25,30], probabilistic collocation approaches [2,17,21], and stochastic finite elements methods [6,12,13,22,29]. In Monte Carlo simulation [9] many deterministic problems are solved for randomly varying parameter values. Non-intrusive polynomial chaos methods [10,20] attempt to reduce the number of deterministic solves by using a polynomial interpolation of the samples in parameter space. An effective sampling in suitable Gauss quadrature points is employed in probabilistic collocation approaches [2,17,21]. A more robust approximation is achieved by stochastic finite elements methods [6,12,13,22,29], in which a piecewise polynomial interpolation of the samples is employed. All these methods have in common that they are mainly developed for steady problems such as in [26], although unsteady applications can be found in, for example, [3,14,15,16,19,23].

In unsteady problems non-intrusive uncertainty quantification methods usually require a fast increasing number of samples with time to maintain a constant accuracy. This behavior is caused by the increasing nonlinearity of the response surface for increasing integration times [22]. This effect is especially profound in problems with oscillatory solutions in which the frequency of the response is affected by the random parameters [18,19,23]. The frequency differences between the realizations lead to increasing phase differences with time, which in turn result in an increasingly oscillatory response surface and more samples. Asymptotic behavior is of practical interest in, for example, post-flutter analysis of fluid–structure interaction systems [3]. Resolving the effect of random input parameters in these long time integration problems requires a large number of deterministic computations. Especially in computationally intensive unsteady flow computations and fluid–structure interaction simulations, such a large sample size can lead to impractically high computational costs. For applications involving oscillatory motion a Fourier chaos basis has been introduced by Millman et al. [14]. In this paper, an efficient alternative approach for uncertainty quantification in oscillatory problems is proposed.

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The probabilistic collocation for limit cycle oscillations (PCLCO) [27] approach was developed by the authors to achieve a constant accuracy in time with a constant number of samples for period-1 oscillations. The PCLCO formulation is based on the application of probabilistic collocation [2,17,21] to a time-independent parametrization of the sampled periodic responses instead of to the time-dependent samples themselves. Due to the time-independent parametrization the interpolation accuracy of PCLCO is independent of time for a constant number of samples. The parametrization employed in PCLCO consists of the frequency, relative phase, amplitude, a reference value, and the normalized period shape. The applications of PCLCO were, however, limited to the asymptotic range of period-1 oscillations with a sufficiently smooth response surface as function of a single random input parameter.

The range of applicability of PCLCO was extended in [28] to the unsteady adaptive stochastic finite elements formulation based on time-independent parametrization (UASFE-ti), which combines a robust adaptive stochastic finite elements (ASFE) interpolation based on Newton–Cotes quadrature in simplex elements [29] with an extension of the time-independent parametrization of PCLCO. The robust ASFE interpolation enables resolving singular behavior in the system response such as bifurcation phenomena. The time-independent parametrization is in UASFE-ti extended to (1) non-periodic oscillations by accounting for a damping factor and (2) higher-period oscillations by adding an algorithm for identifying higher-periods. UASFE-ti has also successfully been applied to problems with multiple random input parameters. The UASFE-ti formulation can, however, be subject to a considerable parametrization error in the time-independent representation of the sampled time series. Furthermore, the application of UASFE-ti is limited to the asymptotic range of responses which allow for a time-independent parametrization.

In this paper, an alternative unsteady adaptive stochastic finite elements formulation based on interpolation at constant phase (UASFE-cp) is developed to further improve the accuracy and extend the applicability of UASFE-ti. As mentioned above, the usual increase of the number of samples with time is caused by increasing phase differences between the realizations. Scaling the samples with their phase and performing the uncertainty quantification interpolation of the samples at constant phase instead of at constant time, eliminates the effect of the phase differences. The increase of the number of samples with time due to an increasingly oscillatory response surface is, therefore, avoided by interpolation at constant phase. In addition to the constant number of samples in time, the proposed UASFE-cp formulation has the following three advantages over UASFE-ti:

(1) *Parametrization error is eliminated*
The time-independent parametrization of the samples in UASFE-ti is subject to numerical discretization and interpolation errors. UASFE-cp uses an exact representation of the samples, which improves the convergence behavior of the method.
(2) *Time-dependent functionals can be resolved*
The application of UASFE-ti is limited to time series which can be represented by time-independent functionals such as frequency and damping. UASFE-cp is applicable to time series in which these functionals change in time. Time-dependent functionals are encountered in practice in, for example, damped nonlinear systems.
(3) *Transient behavior can be captured*
Deterministic transient behavior is an important special case of time-dependent functionals that cannot be captured by the time-independent parametrization of UASFE-ti. The UASFE-cp formulation is capable of resolving the effect of random parameters in both the asymptotic and transient regime of the samples. Transient behavior is seen in virtually all nonlinear practical applications.

UASFE-cp can be applied to problems in which the phase of the oscillatory samples is well-defined. Multi-frequency signals can be handled by first decomposing them into multiple single frequency time series using a wavelet decomposition. The unsteady adaptive stochastic finite elements formulation based on interpolation at constant phase is introduced in Section 2. The effect of the elimination of the parametrization error on the convergence of UASFE-cp is studied for a mass–spring–damper system in Section 3.1. In Section 3.2 UASFE-cp is employed to resolve the effect of multiple random parameters on a response with time-dependent functionals of the damped nonlinear Duffing oscillator. The stochastic bifurcation behavior of the fluid–structure interaction system of nonlinear flow around an elastically mounted airfoil with nonlinear structural stiffness is analyzed in Section 3.3. This application involves transient behavior in the post-bifurcation region. Results for various probability distributions are compared to those of UASFE-ti and Monte Carlo simulations. The paper is concluded in Section 4.

2. Unsteady adaptive stochastic finite elements based on interpolation at constant phase

The procedure for interpolation at constant phase in the unsteady adaptive stochastic finite elements framework is developed in Section 2.1. The adaptive stochastic finite elements formulation employed for the interpolation is briefly reviewed in Section 2.2. In Section 2.3 the resulting UASFE-cp algorithm is summarized.

2.1. Interpolation at constant phase

Consider a dynamical system subject to n uncorrelated second-order random input parameters $\mathbf{a}(\omega) = \{a_1(\omega), \ldots, a_n(\omega)\} \in A$, which governs an oscillatory response $u(\mathbf{x}, t, \omega)$

$$\mathscr{L}(\mathbf{x}, t; u(\mathbf{x}, t, \omega)) = S(\mathbf{x}, t), \tag{1}$$

with operator $\mathscr{L}$ and source term S defined on domain $D \times T$, and appropriate initial and boundary conditions. The spatial and temporal dimensions are defined as $\mathbf{x} \in D$ and $t \in T$, respectively, with $D \subset \mathbb{R}^d$, $d = \{1, 2, 3\}$, and $T = [0, t_{\max}]$. A realization of the set of outcomes Ω of the probability space $(\Omega, \mathscr{F}, P)$ is denoted by $\omega \in \Omega$, with $\mathscr{F} \subset 2^\Omega$ the σ-algebra of events and P a probability measure.

Assume that the phase of the oscillatory samples $u_k(t) \equiv u(t, \omega_k)$ for realizations of the random parameters $\mathbf{a}_k \equiv \mathbf{a}(\omega_k)$ is well-defined for $k = 1, \ldots, N_s$. The argument $\mathbf{x}$ has been dropped here for convenience in the notation. In order to interpolate the samples $u_k(t)$ at constant phase, first, their phase as function of time $\phi_k(t)$ is extracted from the deterministic solves $u_k(t)$. Second, the time series for the phase $\phi_k(t)$ are used to transform the samples $u_k(t)$ to functions of their phase $u_k^*(\phi_k)$ instead of time, see Fig. 1. For discrete time histories the vectors u_k and u_k^* are identical. Third, the transformed samples $u_k^*(\phi_k)$ are interpolated to the function $u^*(\phi, \omega)$ using adaptive stochastic finite elements interpolation. This step involves both the interpolation of the sampled phases $\phi_k(t)$ to the function $\phi(t, \omega)$ and the interpolation of the samples $u_k^*(\tilde{\phi})$ to the function $u^*(\tilde{\phi}, \omega)$ at constant phase $\tilde{\phi}$. Repeating the latter interpolation for all phases $\tilde{\phi}$ results in the function $u^*(\phi, \omega)$. Finally, transforming $u^*(\phi, \omega)$ back to $u(t, \omega)$ using $\phi(t, \omega)$ yields the unknown response surface of the system response as function of the random parameters $\mathbf{a}(\omega)$ and time t. Integrating this response surface approximation results in an approximation of the statistical moments of the response.

The phase $\phi_k(t)$ is extracted from the samples based on the local extrema of the time series $u_k(t)$. A trial and error procedure