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High accuracy numerical investigation of double-diffusive convection in a rectangular cavity under a uniform horizontal magnetic field and heat source

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ABSTRACT

Double-diffusive convection flows of a binary mixed electrically conducting fluid in the presence of a uniform horizontal magnetic field and heat source are investigated numerically in a rectangular cavity with the upper and lower walls being insulated and impermeable and the left and right walls being constant temperatures and concentrations. A high accuracy compact scheme, which is fourth-order accuracy in space and third-order accuracy in time, is applied to solve the problems based on the stream function-vorticity formulation of Navier-Stokes equation. Numerical simulations are carried out in a wide range of Hartmann number (*Ha*), Lewis number (*Le*), Rayleigh number (*Ra*) and the heat generation or absorption coefficient (ϕ) at the Prandtl number $Pr = 0.025$ for the electrically conducting fluid such as molten gallium in the rectangular cavity with the aspect ratio 2. The computed results show that the oscillatory behavior would disappear with the increase of the strength of the magnetic field, and the total kinetic energy in the cavity is inhibited proportionally to $(1-\lambda)^2 Ra^2 Ha^{-\beta}$, where β is between 3.5 and 4, under the strong magnetic field and weak heat source. In addition, asymptotic solutions of the average Nusselt and Sherwood numbers on the left and right walls in the presence of very strong magnetic field, which only dependent on ϕ, are deduced and proved by the present numerical results.

1. Introduction

In the recent decades, laminar double-diffusive convection of electrically conducting fluids in the presence of a magnetic field has been one of the major interesting research subjects due to its widely scientific and engineering applications in such practical problems as cooling of electrical equipment, nuclear reactor, transport of contaminant, geology and chemical process, and astrophysics and heat exchanger. Since the velocity, the temperature and the concentration governing equations are coupled due to the buoyancy force, and the Lorentz force driven by the magnetic field is connected with the velocity, the study of the magnetohydrodynamics (MHD) double-diffusive convection is complicated.

Simpler than the MHD double-diffusive convection, the MHD natural convection which stands for the laminar natural convection of a single electrically conducting fluid in the presence of magnetic field has been investigated by many researchers using experimental [1], analytical [2,3] methods in the early years. In the recent decades, although still some scholars have extended some analytical methods to the more complicated problems such as a second grade fluid [4] for the MHD natural convection, many numerical results have been reported by means of various numerical methods. For example, Ozoe and his collaborators [5,6] investigated numerically the natural convection of an electrically conducting fluid ($Pr = 0.054$) in the presence of magnetic field using two dimensional (2D) and three dimensional (3D) models. Rudraiah et al. [7] developed a finite difference scheme to solve the MHD natural convection at $Pr = 0.733$, and found the average Nusselt number decreases with an increase of Hartmann number. Ben Hadid et al. [8,9] solved the problems by combining a second-order central differentiation and a fourth-order compact Hermitian method in the 2D and 3D cases to validate their analytical solutions for the central region and for the turning flow region under some assumptions. Al-Najem et al. [10] employed a control volume algorithm to numerically investigate the laminar natural convection in tilted enclosure with transverse magnetic field. A SIMPLER algorithm was used by Pirmohammadi [11] for laminar

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Nomenclature

A	aspect ratio
$\vec{B}$	magnetic field vector
b_0	inducted magnetic field vector
C	concentration
C_h	high species concentration
C_l	low species concentration
C_p	specific heat at constant pressure
E	total kinetic energy
$\vec{E}$	electric field vector
g	gravitational acceleration
h	spacial grid size
H	cavity height
Ha	Hartmann number
$\vec{J}$	electric current density vector
Le	Lewis number
Nu	average Nusselt number
p	pressure
Pr	Prandtl number
Q_0	dimensional heat generation or absorption coefficient
Ra	Rayleigh number
Re_m	magnetic Reynolds number
Sh	average Sherwood number
t	time
T	temperature
T_h	high wall temperature
T_l	low wall temperature
u	horizontal velocity component
v	vertical velocity component
$\vec{V}$	velocity field vector
W	cavity width
x	horizontal coordinate
y	vertical coordinate

Greek symbols

β	index for Hartmann number in fitted power law of total kinetic energy
β_C	compositional expansion coefficient
β_T	thermal expansion coefficient
κ_C	species diffusivity
κ_T	thermal diffusivity
λ	buoyancy ratio
ν	kinematic viscosity
ω	vorticity
ϕ	dimensionless heat generation or absorption coefficient
ψ	streamfunction
ρ	fluid density
σ_e	electrical conductivity

natural convection flows in the presence of a longitudinal magnetic field at $Pr = 0.733$ and extended to a tilted square enclosure [12]. Recently, Lo [13] proposed a high-resolution differential quadrature method to simulate the effect of a transverse magnetic field on buoyancy-driven magnetohydrodynamic flow in an enclosure and concluded that the heat transfer rate is at its maximum for higher Pr and in the absence of MHD effects. Very recently, Yu et al. [14] numerically investigated the problems with $Pr = 0.025$ using a second-order compact scheme based on the stream function-velocity formulation of Navier-Stokes equation and discussed the effects of three parameters including the Hartmann number, the direction of the magnetic field and the aspect ratio of the cavity.

Because the MHD double-diffusive convection is more complicated, now it has been investigated mostly using numerical methods. Chamkha and Al-Naser [15] first investigated unsteady heat and mass transfer by natural convection flow of a heat-generating and electrically conducting fluid inside a rectangular enclosure in the presence of a transverse magnetic field, using a second-order central difference methodology based on the stream function-vorticity formulation of Navier-Stokes equations. In their model, the constant temperatures and concentrations are imposed along the left and right walls of the enclosure, while the top and bottom walls are assumed to be adiabatic and impermeable to mass transfer. In that paper, the authors discussed the effect of the magnetic field and the heat source, and found that the periodic oscillatory behavior in the stream function inherent in the problem was decayed by the presence of the magnetic field and this decay in the transient oscillatory behavior would be speeded up by the presence of a heat source. Later, Ma [16] developed a temperature-concentration lattice Bhatnagar-Gross-Krook (TCLBGK) model, with a robust boundary scheme for simulating the same problem again, and the numerical results were found to be in good agreement with those of previous studies [15]. Meanwhile, Teamah [17] also solved the same problems with a wider range of the parameters using a central finite difference method based on the primitive variables formulation of the steady Navier-Stokes equations to analysis the effect of thermal Rayleigh number, heat generation or absorption coefficient and Hartmann number. In that research, the author pointed out that the average Nusselt and Sherwood numbers would have constant values over a range of thermal Rayleigh number for $Ha > 20$, and this range increases with increasing Hartmann number. Recently, Teamah et al. [18,19] extended their previous work to study the MHD double-diffusive convection problem in an inclined rectangular enclosure and discussed the effects involving the inclination angle, Hartmann number, Rayleigh number and heat generation or absorption coefficient. Besides the above work, there are some similar work for the MHD double-diffusive convection but for different boundary conditions or different heat sources. For example, Chamkha and Al-Naser [20] considered another MHD double-diffusive convection with the constant heat and mass fluxes along the left and right walls of the enclosure. Borjin et al. [21] focused on the semitransparent fluid in the MHD double-diffusive convection, in which the source terms of the heat source had a special form. Rahman et al. [22] investigated the conjugated effect of joule heating MHD double-diffusive mixed convection in a horizontal channel with an open cavity. In their research, the boundary conditions and the heat source were slightly different from the original model proposed in [15]. In addition, Maatki et al. [23] presented a numerical study of a double-diffusive convection in a cubic cavity filled with a binary mixture without heat source to discuss the influence of the magnetic field on the structure of the three-dimensional flow. Very recently, some scholars began to take the numerical experiments to study the magnetohydrodynamic double-diffusive natural convection in trapezoidal cavities [24,25].

For the previous studies, the range of the dimensionless parameters are listed in Table 1. It can be seen that although many scholars have carried out the parametric study of the MHD double-diffusive convection, the effects of some parameters still need to be investigated. For example, the Lewis number in those papers was always be fixed at 1 or 2, however, this number is an interest-