International Journal of Heat and Mass Transfer 110 (2017) 768–777

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Steady state and transient analytical modeling of non-uniform convective cooling of a microprocessor chip due to jet impingement

S. Luhar, D. Sarkar, A. Jain *

Mechanical and Aerospace Engineering Department, University of Texas at Arlington, USA

ARTICLE INFO

Article history:
Received 23 December 2016
Received in revised form 10 February 2017
Accepted 17 March 2017

Keywords:
Jet impingement cooling
Thermal management
Thermal conduction
Convection
Analytical methods

ABSTRACT

Heat removal from microprocessor chips with multiple regions of dynamic heat generation remains a critical technological challenge. Excessive temperature rise is undesirable for performance as well as reliability. Jet impingement cooling has been widely investigated as a potential thermal management technique due to the capability of localized cooling and of dynamically following the heat generation distribution. A jet offers large local convective heat transfer coefficient, for which theoretical models and correlations have been proposed for a variety of scenarios. However, not much work exists on using this information to determine the resulting temperature distribution. This paper addresses this need by developing analytical steady state and transient heat transfer models that account for the spatial variation in convective heat transfer coefficient and for spatially non-uniform heat flux. The solution is derived in the form of an infinite series, the coefficients of which are determined by solving a set of algebraic equations. Temperature rise predicted by the models are found to be in excellent agreement with finite-element simulations, while offering faster computation time and easier integration with design and performance optimization tools used in microelectronics. The analytical model is used for predicting temperature rise in a variety of scenarios to examine interesting optimization problems such as the cooling of multiple hotspots with a single jet, determining the optimal location of a jet, etc. Results presented here may facilitate improved thermal design and real-time performance optimization of microprocessor chips.

Overbars represent variables in Laplace domain.

1. Introduction

Cooling of a microprocessor chip is an important technological problem that has attracted significant research over past several decades [1–4]. Heat generated during transistor operation on a chip must be conducted through the chip and package, and rejected to the ambient in order to maintain the microprocessor temperature below an acceptable threshold. Thermal management directly affects device performance, as the mobility of charge carriers deteriorates at higher temperatures [5]. Device and package reliability is also adversely affected by high temperatures. Most modern microprocessor chips are multi-core in nature, and include several other power-intensive blocks such as Graphics Processing Units (GPUs) on the same substrate. This results in multiple regions of high power density, or hotspots, on the chip. Further, hotspots also shift dynamically, depending on the nature of microprocessor load, thereby presenting significant thermal management challenges. Specifically, it is difficult to reduce peak temperature rise on a hotspot using a passive thermal management technique that does not specifically address the hotspot location and the dynamic changes in power dissipation on the chip.

Natural convective cooling may be sufficient for very low power chips. At higher powers, air cooling is employed, typically by attaching a metal heat sink to the chip via a thermal interface material and heat spreader, and providing air flow over the heat sink [6]. Heat removal is also often carried out using a heat pipe or vapor chamber [7,8], particularly in space-constrained applications such as laptops. Single-phase and two-phase liquid cooling offer much larger heat transfer coefficients than air cooling. Much research has also been carried out for investigating liquid cooling for thermal management of higher-power chips [9]. These include liquid flow through microchannels, either in the heat sink, or on the back of the microprocessor chip itself, jet impingement on the chip backside [10], thin film liquid cooling utilizing electrowetting-on-discharge (EWOD) [11], etc. Both implementation and modeling of liquid-based cooling are more complicated than air cooling.

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* Corresponding author at: 500 W First St, Rm 211, Arlington, TX 76019, USA.
E-mail address: jaina@uta.edu (A. Jain).

http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.03.064
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Nomenclature

a,b,c	dimensions
$A_{00}, B_{00}, A_{nm}, B_{nm}$	coefficients in the temperature field solution
C_{00}, C_{nm}	Fourier series coefficients for temperature field
C_p	heat capacity
d	jet diameter
h	convective heat transfer coefficient
k	thermal conductivity
N	norms
P_{00}, P_{nm}	Fourier series coefficients for heat flux field
q	heat flux
r	distance away from jet center
s	Laplace parameter
t	time
T	temperature rise
α	thermal diffusivity
γ	parameter representative of the width of the h vs r curve
λ	eigenvalue
ρ	density

Subscripts

max	maximum
min	minimum
x,y,z	rectilinear coordinates

A laminar liquid jet impinging on the backside of a chip offers very large local convective heat transfer coefficients in the vicinity of the impingement spot. This cooling approach offers several advantages such as spatially directed cooling, and rapid temporal response, and thus has been extensively studied. Key challenges in this approach include management of vapor formation due to boiling, laminar fluid delivery and exit, and dynamic hotspot management. Several papers have demonstrated the experimental implementation of this approach, often employing a chip with resistive heating and temperature sensors for mimicking an actual microprocessor chip. Synthetic air jets impinging on such a thermal test die have been shown to result in significant reduction in thermal resistance [12]. A method has been developed for three-dimensional visualization of single and multijet arrays using micron resolution particle image velocimetry [13]. Experiments have been carried out to study the effect of jet impingement of alumina–water based nanofluids for a range of physical parameters such as Reynolds number, Prandtl number and volume fraction [14]. Cu–water nanofluid jet array impingement has been reported to result in 6.8% heat transfer enhancement [15]. In comparison with a sizable literature on experimental investigation, there is relatively lesser work done on theoretical modeling of jet impingement based cooling of microprocessors.

A key parameter to consider in such a modeling effort is the spatial variation of convective heat transfer coefficient due to the impinging jet. Correlations for different shapes and flow conditions have been developed through experiments and theoretical modeling. Typically, the heat transfer coefficient is the highest in the vicinity of the impingement spot, and reduces farther away. A number of heat transfer regions have been identified, in which heat transfer correlations have been developed. Analytical development of correlations for local Nusselt number for single phase free surface circular liquid jets has been carried out [16]. By combining experimental results and theoretical solutions of jet impingement boundary layer, the impinging jet has been shown to hydrodynamically evolve through four distinct regions: stagnation zone, boundary layer, viscous similarity, developing turbulence and fully turbulent [17].

While such models help understand the fundamental nature of heat transfer in an impinging jet, such models have not been sufficiently translated into tools for predicting temperature distribution in the presence of an impinging jet. Modeling the spatial variation in convective heat transfer coefficient due to an impinging jet presents significant analytical difficulties that are not present when the heat transfer coefficient is uniform [18]. Some work exists where spatially varying convective heat transfer has been accounted for in fins [19], heat generating slab [20], cylinder [21] and sphere [22] using a variant of Fourier series expansion method, but there is a lack of such work for jet impingement cooling of microprocessors.

In this paper, a theoretical model is developed for predicting the steady-state and transient temperature distribution on a microprocessor chip in presence of spatially varying convective cooling due to jet impingement. A series solution is derived, and it is shown that the coefficients in this series can be determined by solving a set of coupled algebraic equations. The transient problem is solved by combining this approach with the Laplace transform technique. The resulting solutions are shown to agree well with finite-element simulation results. The models are used for analyzing the effect of jet cooling on thermal performance of microprocessor chips. Several interesting optimization problems, such as jet placement and jet fluid distribution are analyzed using the model, demonstrating the capability of rapid computation of temperature rise in a microprocessor with spatial and dynamic variation of convective cooling and heat generation.

2. Derivation of temperature distribution in steady state

Fig. 1 shows a schematic of the geometry of a microprocessor chip of dimensions $a \times b \times c$, with spatially and temporally varying heat flux on the bottom face and spatially varying convective heat transfer coefficient $h(x,y)$ on the top face due to impingement of one or multiple jets. The steady state problem is considered in this section. In general, thermal conductivity is assumed to be orthotropic. In this case, the governing energy equation for the temperature field is given by

$$k_x \frac{\partial^2 T}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2} + k_z \frac{\partial^2 T}{\partial z^2} = 0 \tag{1}$$

The temperature field $T(x,y,z)$ satisfies the following boundary conditions given by

$$\frac{\partial T}{\partial x} = 0 \quad \text{at } x = 0, a \tag{2}$$

$$\frac{\partial T}{\partial y} = 0 \quad \text{at } y = 0, b \tag{3}$$

$$k_z \frac{\partial T}{\partial z} + q(x,y) = 0 \quad \text{at } z = 0 \tag{4}$$

$$k_z \frac{\partial T}{\partial z} + h(x,y) \cdot T = 0 \quad \text{at } z = c \tag{5}$$

In this case, the solution for the temperature field may be written as the following two-variable Fourier series [23,24]

$$T(x,y,z) = C_{00}(z) + \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} C_{nm}(z) \cos\left(\frac{n\pi x}{a}\right) \cos\left(\frac{m\pi y}{b}\right) \tag{6}$$

Note that the double summation in Eq. (6) and subsequent equations in Sections 2 and 3 excludes the case where n and m are both