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Technical Note

Effect of surface heat dissipation on thermocapillary convection of low Prandtl number fluid in a shallow annular pool

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ABSTRACT

In order to understand the effect of surface heat dissipation on thermocapillary convection of low Prandtl number in a shallow annular pool, a series of three-dimensional numerical simulations were carried out by using the finite volume method. Results indicate that with the increase of surface heat dissipation, the thermocapillary convection intensity increases at first, and then decreases slightly; furthermore, the center of the thermocapillary convective cell moves gradually to the hot outer wall. With the increase of Marangoni number, the stable axisymmetric flow bifurcates orderly to the three-dimensional steady flow and the three-dimensional oscillatory flow at a weak surface heat dissipation. However, it bifurcates directly to the three-dimensional oscillatory flow at a strong heat dissipation. After thermocapillary convection destabilizes, the hydrothermal waves appear at the weak surface heat dissipation, but the longitudinal rolls near the outer wall are dominant at the strong heat dissipation. The flow pattern transition is always accompanied by the variations of the temperature fluctuation amplitude, the oscillatory frequency and the wave number.

1. Introduction

In the past few decades, thermocapillary convection has received much attention from both fundamental and industrial aspects. Many scholars have been focused on thermocapillary convection of the liquid pool with an adiabatic free surface, and achieved fruitful results [1–3]. Smith and Davis [4] studied thermocapillary convection instabilities by the linear stability analysis for the liquid layer subjected to an imposed lateral temperature gradient. They found that there are two types of three-dimensional instabilities, i.e. stationary longitudinal rolls and oblique hydrothermal waves (HTWs) depending on Prandtl (Pr) number of the fluid and the basic flow pattern. Li et al. [5–8] performed a series of three-dimensional numerical simulations of thermocapillary convection for the fluids with low ($Pr = 0.011$) and moderate ($Pr = 6.7$) Prandtl numbers in the annular liquid pool and analyzed the physical mechanism of the hydrothermal wave formation in the shallow liquid pool. Schwabe et al. [9,10] carried out the experimental determination of the critical condition of the thermocapillary convection destabilization for the moderate Prandtl number fluid in an annular pool, and then explored various flow patterns of oscillatory thermocapillary convection by direct numerical simulations. Furthermore, the stationary longitudinal roll pattern and the HTWs in an annular pool were also observed by many experiments [11–14].

It must be pointed out that the previous researches are conducted without considering heat dissipation from the free surface. As a matter of fact, due to the non-equilibrium effect on the free surface, surface heat dissipation is inevitable and is also a common thermal process in engineering fields, which has significantly effect on thermocapillary convection of the liquid layer. However, thermocapillary convection with surface heat dissipation in an annular pool subjected to a horizontal temperature gradient is few investigated. Surface heat dissipation is bound to change the temperature distribution along the free surface and impacts on the flow patterns and the critical value of the flow bifurcation. There are many kinds of surface heat dissipation, such as convective and radiative heat transfer, and the evaporative cooling. Jing et al. [15] performed three-dimensional numerical simulations of $LiNbO_3$ melt flow in an open crucible with the side wall heated at constant heat flux and the adiabatic bottom to reveal the mechanism of the well-known surface spoke patterns when the radiation heat loss from the melt surface to the ambient was considered. They certified that the spoke patterns are caused by the Marangoni instability in the thin thermal boundary layer near the free surface. On the contrary, a constant temperature boundary condition at the crucible side

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wall stabilizes the melt flow and the generation of the spoke pattern becomes very difficult. Zhang and Chao [16] carried out the experimental observations on thermal convection that is caused by the evaporation on the thin liquid layer in the rectangular cavity. It was found whether the liquid layer at the bottom is heated or cooled, as long as the evaporation on the free surface is strong enough, thermal convection will happen in the fluid layer. In this case, liquid surface evaporation is the driving force of Marangoni-Bénard convection. Zhu and Liu [17,18] made experimental observations and numerical simulations on thermocapillary convection of the evaporating liquid layer in the rectangular pool subjected to a horizontal temperature gradient. The results show that the interfacial evaporation has a great influence on the instability of thermocapillary convection, and the evaporation intensity is related to the non-equilibrium degree through the evaporation interface, which depends on the evaporation Biot (Bi) number on the free surface. Hoyas et al. [19,20] studied the instabilities which appear in a cylindrical annulus with the heating bottom and the opening free surface to the atmosphere by means of the linear stability analysis. It was found that Biot number determines the shape of the growing bifurcation at $Pr > 10$. However, at $Pr < 2$, the main parameter controlling the shape of the growing bifurcation is Prandtl number. After the flow destabilization, whether the various flow patterns including stationary rolls, hydrothermal waves and longitudinal rolls, appear or not, depends mainly on Biot number, Bond number and Prandtl number. Doumenc et al. [21] and Touazi et al. [22] carried out a linear stability analysis on the transient Rayleigh-Bénard-Marangoni convective instability due to surface cooling induced by solvent evaporation when both thermocapillarity and buoyancy forces are taken into account. They predicted the critical Marangoni and Rayleigh numbers, and the critical wave number in a large range of Biot and Prandtl numbers.

As mentioned above, the existing researches mostly focused on Bénard-Marangoni convection or Rayleigh-Bénard-Marangoni convection of a liquid layer with a vertical temperature gradient. Although a linear decreasing temperature profile from the inner to the outer cylinder in the annular pool was considered in the works of Hoyas et al. [19,20], however, both two lateral cylinders are assumed to be adiabatic. In order to extend the existing knowledge, this paper presents a series of three-dimensional numerical simulations on thermocapillary convection of the low Prandtl number in a shallow annular pool with different specified temperatures at the inner and outer cylinders when surface heat dissipation is taken into account.

2. Problem statement

A shallow annular pool with inner radius r_i, outer radius r_o and depth d is filled with the low Prandtl number fluid of $Pr = 0.011$. The radius ratio of the annular pool is defined as $\eta = r_i/r_o$ and the aspect ratio as $\varepsilon = d/(r_o - r_i)$, respectively. The inner and outer cylinders are heated and cooled at specific temperatures T_i and T_o, and the bottom is considered to be thermally adiabatic. On the free surface, the thermocapillary force and the heat dissipation are taken into account.

For simplicity, it is reasonable to introduce the following assumptions: (1) fluid is an incompressible Newtonian fluid, whose physical properties are mostly considered as constant except the surface tension; (2) the velocity is small and the flow is laminar; (3) the free surface is flat and non-deformable; (4) the ambient temperature above the free surface is the same with the inner cylinder temperature T_i. With the above assumptions, the mathematical model can be expressed by the following dimensionless mass, momentum and energy equations, in which $(r_o - r_i)$, $\nu/(r_o - r_i)$, $(r_o - r_i)^2/\nu$ and $\mu\nu/(r_o - r_i)^2$ are used as scale quantities for length, velocity, time and pressure, respectively.

$$\nabla \cdot \boldsymbol{V} = 0 \tag{1}$$

$$\frac{\partial \boldsymbol{V}}{\partial \tau} + \boldsymbol{V} \cdot \nabla \boldsymbol{V} = -\nabla P + \nabla^2 \boldsymbol{V} \tag{2}$$

$$\frac{\partial \Theta}{\partial \tau} + \boldsymbol{V} \cdot \nabla \Theta = \frac{1}{Pr} \nabla^2 \Theta \tag{3}$$

Here $\Theta = (T - T_i)/(T_o - T_i)$. $\boldsymbol{V}$ is the dimensionless velocity vector, and τ and P are the dimensionless time and pressure, respectively.

To begin with, the fluid is assumed to be motionless and in thermal equilibrium with the environment above the free surface. Therefore, the initial conditions are

$$\tau = 0,\ U = V = W = 0,\ \Theta = 0. \tag{4a-b}$$

The no-slip boundary condition is applied to all the solid walls. The boundary conditions at the free surface are expressed as follows:

$$\frac{\partial U}{\partial Z} = -\frac{Ma}{Pr}\frac{\partial \Theta}{\partial R},\ \frac{\partial V}{\partial Z} = -\frac{Ma}{Pr}\frac{\partial \Theta}{R\partial \theta},\ W = 0,\ -\partial\Theta/\partial Z = Bi\Theta \tag{5a-b}$$

where Ma is Marangoni number, $Ma = \gamma_\Gamma \Delta T(r_o - r_i)/(\mu\alpha)$, γ_Γ surface tension temperature coefficient, μ dynamic viscosity. Bi is the integrated Biot (Bi) number, $Bi = h(r_o - r_i)/\lambda$. h is the surface heat transfer coefficient, which includes the single-phase convective, radiative and evaporative heat transfer from the free surface to the environment.

The finite volume method is used to discretize the dimensionless governing equations. The central difference approximation is applied to the diffusion terms while the QUICK scheme is used for the convective terms and the SIMPLE algorithm for correcting simultaneously the pressure and the velocity. The dimensionless time step ranges from 0.11×10^{-5} to 0.37×10^{-5}. If the maximum relative error of all these fundamental equations among the computational domain gets below 10^{-4} at each time step, the solution is considered to be convergent.

Non-uniform staggered grid of $60^R \times 22^Z \times 240^\theta$ is applied, which is encrypted near the solid walls and the free surface; the mesh convergence of all the grids is checked. In order to validate the current numerical scheme, we reproduced numerically the flow pattern that has been experimentally observed by Azami et al. [23]. Then, the simulation has been carefully done under the same condition as the work of Liu et al. [24] when the evaporation on the free surface was considered. It was found that the presented temperature distribution along the free surface is almost the same with the result of Liu et al. [24] when the flow is stable at small Marangoni numbers, as shown in Ref. [25]. These validations provide enough confidence to the accuracy of the numerical scheme.

3. Results and discussion

3.1. Basic flow and stability

In this work, the radius ratio of the annular pool is fixed at $\eta = 0.5$, and the aspect ratios are 0.05 and 0.1. Biot number is limited in $0 \le Bi \le 3$.

When Marangoni number is small, thermocapillary convection is the stable axisymmetric flow, which is called as basic flow. With the increase of Biot number, the increasing heat dissipation on the free surface causes the temperature gradient to decrease and the flow velocity to slow down near the inner wall, which results in that the flow from the hot outer wall begins to return before it