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Oscillation regimes of gas/vapor bubbles

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ABSTRACT

In this work we investigate the effect of heat and mass transfer on the dynamics of gas-vapor bubbles. We present phase diagrams for the bubble oscillation regimes, which are built by comparison of various models with different level of simplification for an air-water system. These diagrams show the range of validity of the simplifying assumptions on the Peclet-number/vapor-content plane, providing an insight on the physical process which regulates the bubble response with respect to external pressure perturbations. The analysis is presented for both the linear and weakly non-linear regime. In the former case we use linearized solutions of the full system; in the latter, numerical simulations validated against the analytical solutions in the linear limit. We show that even at very low frequencies, there exist regimes where transient diffusion effects arise and restrict the applicability of the commonly-adopted assumption of full-equilibrium conditions inside the bubble. Non-linearity is found to restrict even further the range of applicability of this hypothesis, due to the variation of the vapor content beyond a critical value.

1. Introduction

The response of liquids containing bubbles to external pressure changes has important implications in engineering, geophysical and biomedical applications [1,2]. Dilute systems in which bubbles contain a negligible amount of vapor have been extensively investigated, both theoretically and experimentally (see [3] for a review). However, the consequences of phase transition in liquids containing bubbles with an appreciable amount of vapor are not completely understood. Although theoretical and experimental studies show that heat- and mass-transfer effects have a non-negligible influence on the bubble response [4] and thus, on the overall fluid properties [5–7], it is also possible to find conditions where mass-transfer effects are not evident and/or more difficult to capture [8]. Available works in the literature propose different quantities to determine whether mass-transfer effects are relevant or not [9,10] but the problem is that a solid base for modeling still lacks [11]. Systematic approaches for dilute mixtures with bubbles containing vapor in addition to a permanent gas have been proposed only recently [11–13]. In these latter works, the authors firstly address the response of a single bubble to a varying pressure field considering heat- and mass-transfer effects, and then discuss the implications on the speed of sound in the mixture.

The development of numerical codes able to correctly predict the response of bubbles undergoing phase change is challenging. One possibility for fast bubble oscillations is to assume that the influence of both the heat and the mass flux across the interface on the bubble's pressure are negligible compared to the pressure changes imposed by the gas volume change. In this case the bubble response is adiabatic and one can relate volume and pressure changes through a polytropic transformation. At the other extreme, for very slow pressure/temperature variations, it can be assumed that the bubble reaches a thermodynamic equilibrium with its surroundings so that the vapor pressure is uniform and solely given by the system's temperature. Within these two limiting solutions, the mass flux across the interface is influenced by the diffusion of mass and heat both in the liquid surrounding the bubble and inside the bubble playing an important role on how the bubble's pressure change as a function of volume. Unfortunately, the definition of the relevant dimensionless parameters that determine the relevance of various mechanisms on the bubble's response and the total mass flux is not straightforward and it is difficult to find in the literature quantitative studies about the range of validity of various assumptions. In this view, the spherically symmetric assumption provides a simple yet interesting situation to clarify important phenomena about the dynamic response of a single bubble with respect to external pressure/temperature perturbations. Using this framework, several phenomena have already been investigated using simplified models based on the Rayleigh–Plesset equation and an effective equation of state that relates

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the bubble's pressure and volume changes. For instance, it is possible to derive analytical expressions for the resonance frequency and damping factor for pure-gas bubbles oscillating in the linear limit (reviewed in [14]); heat-transfer, rectified diffusion and secondary resonance frequency for pure-vapor bubbles [15–19] and the effect of soluble or insoluble gas on the dynamics of vapor bubbles [20–22]. Fuster & Montel [12] have recently proposed an analytical derivation of the resonance frequency and damping factor for gas-vapor bubbles.

This manuscript presents an analysis of heat- and mass-transfer effects on the dynamic response of gas-vapor bubbles. The manuscript is structured as follows. Firstly, we address the problem of linear oscillations and propose phase diagrams for the bubble oscillation regimes, which are built by the comparison between different simplified models and the full analytical solution for gas-vapor bubbles. These diagrams show the range of applicability of the various simplifying modeling assumptions, providing new insight into the transport phenomena which control the physical response of the bubble with respect to the vapor content and the external pressure perturbation. Firstly, we discuss the regimes on the Peclet-number/vapor-quantity plane for the transfer function (which relates the bubble radius oscillation with the external perturbation) and show that, even for very low frequencies, transient effects can prevent the commonly-adopted assumption of full-equilibrium conditions inside the bubble. Secondly, we explore the regimes beyond the linear limit using numerical solutions. The code, which is validated against the analytical solution in the linear limit, allows us to analyze the response of the bubble for various pressure amplitudes and to show the orbits described by the local quantities on the phase diagrams. Non-linearity is found to restrict the range of applicability of the full-equilibrium assumption when local orbits span into other regimes.

2. Physical model

2.1. Governing equations

We consider a spherically-symmetric, non-reacting, gas-vapor bubble standing in a pure liquid. The model relies on the mass, momentum, energy and species conservation equations [23]. Integrating the mass and momentum equations in the liquid yields the well-known Rayleigh-Plesset equation, which, neglecting the compressibility of the liquid while considering mass transfer effects reads as [24]:

$$R\ddot{R} = -\frac{3}{2}\left(\dot{R}-\frac{J}{\rho_l}\right)^2 + \frac{R\dot{J}}{\rho_l} + 2\frac{J}{\rho_l}\left(\dot{R}-\frac{J}{\rho_l}\right) + \frac{p_b - p_\infty}{\rho_l} - \frac{2\sigma}{\rho_l R} - \frac{4\mu_l}{\rho_l}\frac{\dot{R}}{R}. \tag{1}$$

In the above equation, R is the radius of the bubble, J the vapor-mass flux across the interface, ρ_l the liquid density, p_b the bubble pressure (assumed to be uniform), p_∞ the far-field liquid pressure, σ the surface tension and μ_l the viscosity of the liquid. The internal pressure is assumed to obey the ideal gas law $p_b = \rho_b \mathcal{R}_b T_b$, being ρ_b the density inside the bubble and $\mathcal{R}_b$ the average specific gas constant for the gas/vapor mixture. The energy and species conservation equations in the radial coordinate are:

$$\frac{DT}{Dt} = \frac{1}{\rho c_p}\frac{Dp}{Dt} + \Gamma_t \nabla_r^2 T; \tag{2}$$

$$\rho\frac{DY}{Dt} = \Gamma_m \nabla_r \cdot (\rho \nabla_r Y); \tag{3}$$

where c_p is the specific heat, Y the vapor molar fraction, Γ_t and Γ_m are the thermal and mass diffusivities respectively and r is the radial coordinate. The properties of the gas/vapor mixture are computed from those of the pure substances using an arithmetic average. The total derivative for a generic (scalar or vector) quantity ϕ is defined as $D\phi = \partial_t \phi + v_r \partial_r \phi$, being v_r the radial velocity.

The energy equation (2) is solved both inside the bubble and in the surrounding liquid, while the species conservation equation (3) is solved only for the vapor content inside the bubble, as in this work we neglect the gas solubility in the liquid. We remark as this latter approximation implies to neglect the rectified diffusion due to the gas intake inside the bubble [25,26]; this effect arises in second order and becomes relevant only for very large time scales [27]. In this work we focus on time scales much shorter than those where rectified diffusion effects play a role, which allows us to assume this effect to be negligible.

The radial velocity profile inside the bubble is obtained from the continuity equation, which can be rewritten using the energy equation as [3]:

$$v_b(r) = \frac{1}{\gamma p_b}\left((\gamma-1)\lambda_b \frac{\partial T_b}{\partial r}\bigg|_{r=R} - \frac{1}{3}\dot{p}_b r\right); \tag{4}$$

with γ being the polytropic index (ratio of specific heats) and λ_b the averaged thermal conductivity of the species inside the bubble.

In order to close the problem, an additional equation is required. One possibility is to impose that the interface of the bubble is in equilibrium with the surrounding liquid at every instant. In this case, the vapor concentration is given by the Clausius-Clapeyron equation. Another possibility is to account for kinetic mass transfer effects using the Hertz-Knudsen-Langmuir (HKL) equation, which imposes the mass transfer flux as a function of the difference between the instantaneous vapor pressure and the equilibrium pressure at the interface's conditions. It can be shown that the model accounting for kinetic mass transfer effects converges to the model assuming equilibrium conditions when either vapor diffusion or heat transfer controls the overall mass transfer rate [12]. In this work we focus on these latter conditions, where the kinetics of the phase change does not have important contribution; however, for practical purposes, we retain the HKL model in the implementation of the equations in the numerical code with an accommodation coefficient equal to 0.35 (see [23] for further details).

2.2. Boundary conditions

At the bubble center, the boundary conditions for the energy equation (2) and the species equation (3) are imposed by spherical symmetry as a zero-flux condition:

$$\frac{\partial T_b}{\partial r}\bigg|_{r=0} = 0 \quad \text{and} \quad \frac{\partial Y}{\partial r}\bigg|_{r=0} = 0. \tag{5}$$

At the bubble interface, the temperature profile is assumed to be continuous, so $T_b(r=R) = T_l(r=R)$. The energy balance yields:

$$\lambda_l \frac{\partial T_l}{\partial r}\bigg|_{r=R} = \lambda_b \frac{\partial T_b}{\partial r}\bigg|_{r=R} + JH_v; \tag{6}$$

with H_v being the enthalpy of vaporization/condensation at the interface's temperature. We remark that the mass and momentum conservation at the interface have been directly applied in the derivation of Eq. (1), where the pressure far from the bubble is known. For species, the boundary condition at the interface is given by the continuity of the vapor mass:

$$\rho_b|_{r=R}\Gamma_{m,b}\frac{\partial Y}{\partial r}\bigg|_{r=R} = J(1 - Y|_{r=R}). \tag{7}$$

Finally, in the far field, the liquid temperature is imposed to be equal to the bulk reference temperature T_∞.