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# Constructal optimization of longitudinal and latitudinal rectangular fins used for cooling a plate under free convection by the intersection of asymptotes method

Mehrzad Zadhoush [a], Afshin Ahmadi Nadooshan [b], Masoud Afrand [c,*], Hasan ghafori [a]

[a] Department of Mechanical Engineering, Majlesi Branch, Islamic Azad University, Isfahan, Iran
[b] Department of Mechanical Engineering, Shahrekord University, Shahrekord, Iran
[c] Department of Mechanical Engineering, Najafabad Branch, Islamic Azad University, Najafabad, Iran

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ABSTRACT

The present paper studies the geometric optimization of a fin plate under free heat transfer convection using the constructal theory. In order to check this mode, certain factors are considered, including temperature, heating plate dimensions, and the mass of the materials used for constructing the fins. The used fins are rectangular and the supposed fluid of the environment is air. The main goal here is to determine the role of the arrangement type of the fins in terms of geometric optimization for cooling of the heating plate. In so doing, two types of arrangements have been considered longitudinal and transverse and the solution steps are carried out separately for each arrangement. First, the optimization was studied using the analytical solution via the intersection of asymptotes. Then, in order to validate the analytical method used on the basis of the constructal theory, the numerical method was used for numerical simulation of the work. The results were finally compared and the best and most efficient possible construct for fin arrangement were selected. This optimal geometry in the longitudinal arrangement was achieved with six fins and a heat transfer rate of 34.47 W.



## 1. Introduction

Constructal theory is a new theory for geometric optimization in various subjects such as heat transfer. This theory maintains that the crucial aspect of any system analysis is its physical and geometric construct and that the optimization of this construct is the way to maximize the efficiency of the entire system [1–5].

Unlike thermodynamic optimization, which seeks to establish a balance for minimizing the irreversibility between the heat transfer and the fluid performance, the aim in this theory is to find a comprehensive model to maximize the performance in both of these cases. Various works have been done in the field of geometric optimization using the constructal theory, by trying to focus on geometric parameters to be able to achieve the optimized mode in heat transfer issues [6–8]. The intersection of asymptotes method, first presented by Bejan, has been used to find the optimal point in different problems [9–11]. Heat transfer is one of the cases that can be applied in the constructal theory, including the work of Bejan et al. who studied a vertical wall [12]. They also found a method of heat transfer for maximum density of vertical plates [13]. Zhang et al. analytically and numerically presented the fin arrangements to find the optimal distance between them [14]. Another issue was the thick fins considered for heat sinks [15–17]. Kalbasi and Salimpour compared the maximum temperature with the change in the number of fins for horizontal fins by using phase change materials in a closed environment [18]. Islam et al. studied the effect of the average speed and angle in a long duct [19]. In another study, Linstedt et al. observed three types of fins in a heat sink in the two types of flows laminar and turbulent under convection and thus showed the optimized geometry for them [20]. A similar study was done by Rao et al., who used the teaching-learning-based optimization algorithm for plate-fin heat sinks. Afterwards, they did a simulation of the subject in question via the Ansys 12.1 [21]. In the case of the fins under heat transfer convection, certain other experiments have also been conducted with various other methods, such as sliding-chimney plume and single-chimney plume [22], longitudinal convective-radiative fins [23], perforated fins [24,25], interrupted fins [26], and embossed fin [27]. Li and Byon considered the fins of radial

* Corresponding author at: Department of Mechanical Engineering, Najafabad Branch, Islamic Azad University, Najafabad, Iran.

*E-mail addresses:* masoud.afrand@pmc.iaun.ac.ir, masoud_afrand@yahoo.com (M. Afrand).

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heat sink type and studied the role of the different angles on the Nusselt number and the Elenbass number [28]. They also studied the same issue in order to experimentally and numerically improve the heat transfer properties [29]. Kamkari and Shokouhmand also experimentally tested the role of the phase-change materials in finned and unfinned enclosures for temperature variations [30]. There have also been different studies on longitudinal fins; the geometric parameters have been used in them for constructal optimization so as to improve the heat transfer [31–33]. Karvinen et al. conducted heat flux for a plate fin in both natural and forced convections and aimed to maximize heat dissipation by finding the optimized geometry [34]. Among other applications of the vertical fins is their use in light-emitting diode lamps. Studies have been done on this application. Better cooling of these lamps has a direct impact on their better exposure and therefore better thermal solution [35,36].

In general, much has been done in geometric optimization of the fins and it has been tried to refer to the works conducted in this field.

The works that have been done in this field are applicable for specific status and they do not propose a comprehensive method. Among some of these works, certain relations have been obtained by the use of various methods that these relations lose their proficiency as they change their geometry and do not present a maximum point for heat transfer and the heat transfer increases and decreases uniformly. Hence, we cannot find the optimal geometry by their and thus it is likely that the relationships not to be used [37,38]. The presentation of a comprehensive method for optimized geometry of the fins is among the benefits of the work done. The method of intersection of asymptotes is less addressed and its effects have been studied in heat exchangers [11] or fluid flow in the pipe parallel [39]. However, in this study, we do focus on geometry of the fins and their arrangement on a flat plate.

In this work, the volumes of all fins are fixed together. Hence, with a fixed volume and a geometric change of the fins, the heat transfer would be maximum and this is while in the previous works, the issue of volume has been less addressed. When the thickness of the fins are fixed, as the total volume of fins are fixed, the heat transfer surface would be fixed and the heat transfer coefficient should be maximum and, in fact, by creating restrictions on the geometric parameters, we increased its focus on the performance of the employed method. Among other features of this work is the output of all data under heat flux parameter in order so as to make the comparison of the data tangible and simplifying the comparison of the considered modes.

Given the literature review, the aim of the present study is to investigate the application of the constructal theory in the two types of fins both longitudinal and transverse on a heating plate to achieve maximum heat transfer. By determining the optimized mode in each arrangement, we look forward to finding the final optimized mode between these two arrangements. It should be noted that this work has been done through both analytical and numerical methods.

Another aim of this work is to find optimal height and distance for the fins, wherein the thickness of the lines is considered constant and then it is tried to study the issue aiming at maximum use of the unheated space and improving the heat transfer under the same conditions but with the use of small fins among large fins. In fact, firstly, by studying the first part of this issue, an optimized geometry will be found and then we will try to take a step forward and consider other modes that seem to be more complete and is expected to have a better result. The results of these two modes will be analysed.

## 2. Description of the problem

The main part of this work is done in two stages. Each stage is based on the thermal boundary layer, which is formed on the fins. Newton's law of cooling is used for rating the general heat transfer with the difference that the heat transfer coefficient values in each stage will be achieved by a different method.

The first stage occurs when the formed thermal boundary layer would have no effect on each other. To achieve this mode, the heat transfer coefficient is used by the Ostrach extraction table, where the Nusselt number is provided for air [40]. In the second stage, where the thermal boundary layer overlaps, the heat transfer coefficient is by the relationships that were conducted in an experiment by Jones and Smith and in that they found the relations between the results for heat transfer by changing distance between the fins. They divided the links between the vehicles into two parts based on the distance between the fins, which are explained in the following [41].

After theses stages, a mode will be considered, by which we can maximize the heat transfer by the use of the space between fins and in the second stage to understand if this method can play a role in our main goal, which is finding an optimized geometry. Terms used in this part and the existing parameters will be exactly the same as the previous section. For this part of the work, the work of da Silva et al. was used, where we extracted the relationships to calculate the distance between large and small fins in our work [42]. The achieved optimal work in every section will be ultimately compared with each other and the best geometry will be chosen and introduced. The mentioned relationships in this part will be given fully in the next part.

In the present study, a plate at a high, constant temperature (represented by $T_w$) is investigated. The angle of the base plate with the horizon is zero degree. The aim is to use two fins separately for the plate. The surrounding environment of the plate is assumed to have the constant temperature of $T_\infty$. Keeping the mass (m) and volume (V) of the fins constant, the aim is to find the best geometry. Due to the constancy of mass, if the number of fins is reduced, the distance between them increases. Therefore, to compensate for the shortage and keep the volume constant, the height or thickness of the fins must be increased. Figs. 1 and 2 illustrate the longitudinal and transverse fins used. The number of fins is represented by n and thickness is represented by t.

Figs. 3 and 4 illustrate the final arrangement. Considering fin dimensions, the constant volume v for transverse arrangement equals n.H.W.t. In the longitudinal case, W is replaced by L.

Heat transfer is considered in two limit cases. For instance, in order to compute the optimal distance in the first case, heat transfer between the fins is computed while the fins are at a distance from each other. In the second case, heat transfer is computed while the fins are close to each other.

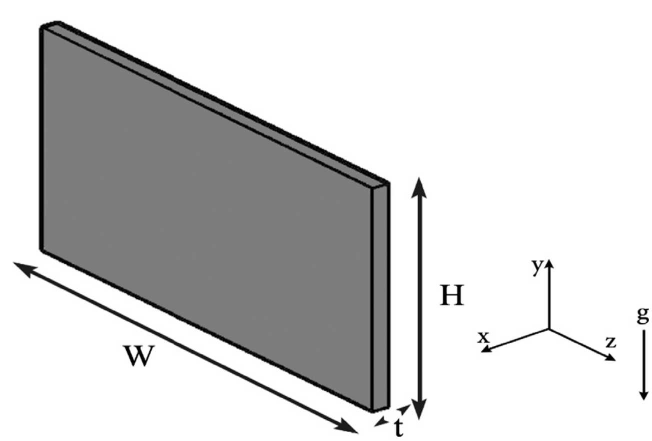


**Fig. 1.** Rectangular latitudinal fin.