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Exact solution for forced convection gaseous slip flow in corrugated microtubes

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ABSTRACT

Forced convection heat transfer in fully developed gaseous laminar slip flow in transversally corrugated micropipes is investigated for first-order slip boundary conditions on the wall. The governing equations subject to first-order slip boundary conditions are solved analytically by means of the epitrochoid conformal mapping. Closed-form exact expressions are obtained for the velocity and temperature fields for the first time. The effects of the corrugation on the pressure drop and heat transfer rate are discussed in detail. Exact results obtained in this paper are compared with the recent approximate analytical solution in the literature, Sadeghi et al. (2011) [1].

1. Introduction

Because of the improvements in manufacturing technology as well as the miniaturization of electronic components due to advances in solid-state electronics, it is now possible to fabricate mechanical parts, for example microtubes, within a micrometer or submicrometer size. Microtubes have been widely used in a large variety of novel applications, such as heat exchangers, Tuckerman [2], microsensors, Cho and Wise [3], Pfahler et al. [4], Pong et al. [5], biological cell reactors, and selective membranes.

Design and optimization of certain microdevices require the analysis of gas flow through microchannels. Because of significant rarefaction effects, the flow physics of gas flow at microscale is quite different from macroscale. The rarefaction effect is measured by the Knudsen number (Kn) defined as the ratio of the molecular mean free path (λ) to the appropriate characteristic dimension of the flow domain. Typical applications in microfluidic systems may involve characteristic dimensions in the range of 10–200 μm, Duan and Muzychka [6]. Based on the degree of rarefaction effect gaseous flow in microchannels may fall in four flow categories: the continuum regime ($Kn \leqslant 10^{-3}$), slip-flow regime ($10^{-3} < Kn \leqslant 10^{-1}$), transition regime ($10^{-1} < Kn \leqslant 10$) and free molecular regime ($Kn > 10$). In the slip-flow regime, the deviation from the continuum behavior is not significant, Renksizbulut et al. [7], Bahrami et al. [8]. Therefore, the standard Navier-Stokes and energy equations can still be employed with the proper boundary conditions accounting for velocity-slip and temperature-jump at the walls Duan and Muzychka [6], Bahrami et al. [8].

Fully developed slip flow and heat transfer in microchannels has been investigated by several authors, Aydin and Avci [9,10] among them. They studied fully developed laminar slip flow forced convection in a micropipe and microchannel between two parallel plates and found that Nusselt number decreases with increasing values of the Knudsen number. Tunc and Bayazitoglu [11] use the integral transform technique to tackle the problem of fully developed laminar slip flow forced convection in a rectangular microchannel under $H2$ type boundary conditions and determine again that the Nusselt number decreases with increasing values of the Knudsen number. Forced convection with slip-flow in a porous parallel-plate microchannel and circular microtube was investigated analytically by Nield and Kuznetsov [12]. They find that velocity slip results in heat transfer enhancement and temperature jump leads to a reduction in heat transfer. Subsequently, Kuznetsov and Nield [13] extended their work to the case of thermally developing forced convection. Analytical solutions of fully developed slip flow forced convection in microchannel with heat sinks

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were presented by Khan and Yovanovich [14] who found that fluid friction decreases and heat transfer increases compared to no-slip flow conditions depending on aspect ratio and Knudsen number, channel size and fluid/wall interaction.

All the works mentioned above deal with smooth geometries, but due to the state of the manufacturing technology it may be impossible to fabricate a smooth channel or tube. Any roughness on the wall will affect both the velocity and temperature fields. Because this effect is more important in corrugated microchannel or microtubes than corrugated macrochannel or tubes it has been the focus of experimental and theoretical studies to model liquid flows in corrugated microtubes. Relevant results in the literature on the impact of corrugations on pressure drop and heat transfer modeling of liquid flows in corrugated microtubes are briefly reviewed hereafter. Peng et al. [15] for instance investigate experimentally the flow characteristics of water flowing through rectangular microchannels having hydraulic diameters of 0.133–0.367 mm and H/W ratios of 0.333–1 and find that in general increasing the ratio H/W increases the friction factor. The reduction of the microchannel hydraulic radius decreases the friction factor significantly for a given H/W. Flow characteristics of water in microtubes were examined by Mala and Li [16]. Their experimental results indicate that for microtubes with smaller diameters the pressure drop is higher than the predictions of the conventional theory. Laminar convective heat transfer and pressure drop of water in 13 different trapezoidal silicon microchannels was investigated experimentally by Wu and Cheng [17]. They determined that Nusselt number and friction coefficient increase with increasing surface roughness and surface hydrophilic property.

Corrugations on channel or tube walls introduce surface-fluid interactions that are not well defined and cannot be easily handled through analytical or numerical treatments. To define the problem theoretically one has to define corrugation. Li et al. [18] propose a model that describes the behavior of rarefied gas flow in long microtubes. The inner surface is modeled as an annulus porous film pressed on an impermeable surface. Duan and Muzychka [19] considered the effects of corrugated roughness on the fully developed laminar flow in microtubes, and obtained an approximate solution in terms of the perturbation parameter taken as the corrugation amplitude to determine the pressure drop in corrugated rough microtubes for continuum no-slip flow and slip flow. Recently, Sadeghi et al. [1] studied the effects of corrugated roughness on the heat transfer characteristics of the fully developed forced convection of a rarefied gas flow through micropipes in slip flow regime with the boundary condition of type $H1$. They used a perturbation approach for both velocity and temperature fields. Their approximate analytical results reveal that corrugated roughness increases both the pressure drop and the heat transfer rate with the magnitude of the increase in the pressure drop exceeding that of the increase in the heat transfer rate.

In this work, forced convection heat transfer in fully developed laminar, rarefied gas flow in transversally corrugated micropipes is considered with the boundary condition of type $H1$ and exact analytical solutions for the velocity and thermal fields are derived. We show that the geometry and the results derived in Sadeghi et al. [1] as well as the results of Duan and Muzychka [6,19] are all special cases of the exact solution presented in this paper.

2. Governing equations and analytical solutions

A thermally and hydrodynamically fully developed flow is considered in this paper. Fluid properties are assumed to be constant thus constitutive parameters do not depend on temperature. Fourier's law of heat conduction is valid and internal energy and thermal conductivity do not depend explicitly on the velocity gradient or other kinematic quantities. With these assumptions the hydrodynamic and thermal problems become fully decoupled. Effects of flow rarefaction at the tube wall on the velocity and temperature fields triggered by first order slip boundary condition at the wall are investigated. The analytical solution for the velocity field under no-slip is developed in Akyildiz et al. [20], and is adapted here to gaseous slip flow.

2.1. Analytical solution for the velocity field

Poiseuille flow of a Newtonian fluid in a tube of average radius a is considered. Due to the symmetries of the problem the velocity field is unidirectional $\boldsymbol{v}^* = [0, 0, w^*(x^*, y^*)]$, and the Navier-Stokes equations collapse onto,

$$\frac{\partial p}{\partial x^*} = 0, \quad \frac{\partial p}{\partial y^*} = 0, \quad \frac{\partial p}{\partial z^*} = \mu\left(\frac{\partial^2 w^*}{\partial x^{*2}} + \frac{\partial^2 w^*}{\partial y^{*2}}\right), \tag{2.1}$$

The star notation indicates dimensional entities. The coordinate axes x^* and y^* are in the cross-sectional plane and the z^* axis points in the longitudinal direction with the origin located at the center of the cross-section. The axial momentum balance $(2.1)_3$ is non-dimensionalized introducing dimensionless variables,

$$x = \frac{x^*}{a}, \quad y = \frac{y^*}{a}, \quad w = \frac{w^*}{w_0} \tag{2.2}$$

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{a^2}{\mu w_0}\frac{dp}{dz} \tag{2.3}$$

The flow domain is mapped onto the domain inside the unit circle via the epitrochoid conformal mapping, Muskheselishvili [21],

$$\zeta = x + iy = \xi + \frac{\varepsilon \xi^{n+1}}{a^n} \tag{2.4}$$

with integer $n \geqslant 1$ and ε representing constants. The transformed domain ξ is defined as,

$$\xi = \rho e^{i\varphi}, \quad 0 \leqslant \rho \leqslant 1, \quad -\pi \leqslant \varphi \leqslant \pi, \quad \rho = \frac{\rho^*}{a}, \tag{2.5}$$

or in terms of Cartesian coordinates x and y,

$$\begin{aligned} x &= \rho\cos\varphi + \varepsilon\rho^{n+1}\cos[(n+1)\varphi] \\ y &= \rho\sin\varphi + \varepsilon\rho^{n+1}\sin[(n+1)\varphi] \end{aligned} \tag{2.6}$$

This map is conformal provided $|\varepsilon|(n+1) < 1$; it maps the inside of the unit circle in the ξ-plane into a compact region in the physical ζ-plane with corrugated boundary of n corrugations. The radial coordinate r associated with the physical plane in Fig. 1 is related to the radial coordinate ρ and the azimuthal angle φ on the unit disk through

$$r^2 = \rho^2 + \varepsilon^2\rho^{2n+2} + 2\varepsilon\rho^{n+2}\cos(n\varphi) \tag{2.7}$$

Under the epitrochoid transformation, Eq. (2.3) and the related boundary conditions become in dimensionless form,

$$\frac{1}{J}\left(\frac{\partial w}{\partial \rho} + \rho\frac{\partial^2 w}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial^2 w}{\partial \varphi^2}\right) = \frac{a^2}{\mu w_0}\frac{dp}{dz} \tag{2.8}$$

$$J = \rho\{1 + \varepsilon^2(n+1)^2\rho^{2n} + 2\varepsilon(n+1)\rho^n\cos(n\varphi)\}$$

$$w(0, \varphi) \neq \infty, \quad w(\rho, 0) = w(\rho, 2\pi/n) = 0, \quad 0 \leqslant \rho \leqslant 1 \tag{2.9}$$

To consider the effects of flow rarefaction at the tube wall, the first-order velocity slip boundary condition is prescribed, Sadeghi et al. [1],

$$w_s = \frac{2 - \sigma_m}{\sigma_m}\lambda_f\left(\frac{\partial w}{\partial n}\right)_w \tag{2.10}$$