International Journal of Heat and Mass Transfer 112 (2017) 918–925

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Thermocapillary effect on the absolute and convective instabilities of film flows down a fibre

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ARTICLE INFO

Article history:
Received 25 February 2017
Received in revised form 8 May 2017
Accepted 8 May 2017

Keywords:
Film flow
Marangoni effect
Absolute and convective instabilities

ABSTRACT

We consider the motion of a gravity-driven flow down a uniformly heated vertical fibre. This flow exhibits rich dynamics including the formation of droplets, or beads, driven by a Rayleigh-Plateau mechanism modified by the presence of gravity as well as the thermocapillarity at the interface. A spatio-temporal stability analysis is performed to investigate the effect of thermocapillarity (Marangoni effect) on the convective/absolute instability (CI/AI) characteristics of the problem. We also performed a numerical simulation of Eq. (30) on the nonlinear evolution of the film to connect the breakup behaviours with the CI/AI characteristics. Our numerical results showed that for various Marangoni number (*Ma*), breakup of the film mainly occurs in the AI regime. With the increase of *Ma*, the film has a tendency to break up into droplets due to the enhancement of the absolute instability.

1. Introduction

The dynamics of a liquid film flowing down a vertical fibre has been encountered in many industrial applications, for example, draining, coating of insulation on a wire, and the protection coating tube walls [1]. It is well known that a cylindrical thread or jet has a tendency to break up into spherical droplets to reduce the surface area due to a surface tension driven mechanism (the Rayleigh-Plateau instability) [2]. For a film flowing down a slender cylindrical fibre, the Rayleigh-Plateau mechanism is modified by the flow driven by gravity. At small Reynolds numbers, the film is always unstable and spontaneously breaks up into a wave train consisting of axisymmetric droplets.

Experimental investigation on the gravity-driven flow on a fibre was performed first by Quéré [3]. The results showed that two different kinds of behaviour can be observed according to the film thickness: For a thick film on a slender fibre, drops develop due to the Rayleigh instability and flow downwards. Some of drops grow by swallowing the other ones, and quickly fall, leaving behind them a thick film which breaks up in turn into droplets. For a thin film on a large fibre, the breakup process may be arrested by the mean flow. The arrest by the mean flow of the latter case was investigated by many authors [4–6] using a lubrication-type (Benney-like) equation for the film thickness wherein the fibre radius a is much larger than the film thickness h.

Kliakhandler et al. [7] studied experimentally the case where the film thickness is larger than the fibre radius. Three qualitatively different regimes of the interfacial patterns in the form of beads were observed in the experiments. In their experiments, the film is at least twice as thick as the fibre radius. Therefore, the previously derived Benney-like equations under the assumption of $h \ll a$ do not apply there. The authors proposed an evolution equation which does not rely on the previously made lubrication-type assumptions. Two typical regimes at relatively small flow rate are well predicted by their model. However, this equation fails to capture a regime that features beads separated by relatively long flat thin-film regions. Craster and Matar [8] derived a new evolution equation similar to that used by Kliakhandler et al. [7] and revisited the same problem. The authors showed that numerical solutions of their model equation yield good agreement with the experimental observations reported by Kliakhandler et al. [7].

All modelling mentioned above are valid for the Reynolds number $Re \sim \mathcal{O}(1)$ or smaller due to the assumption of negligible inertia effects. Ruyer-Quil et al. [9] formulated a two-equation model for the film thickness h and flow rate q using a weighted residuals approach. This model accounting for inertia and streamwise viscous diffusion is valid for both small and $\mathcal{O}(1)$ aspect ratios of h/a. Comparisons between the numerical result and experimental results show good agreement in both linear and nonlinear regimes.

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.05.030

In practical processes, a more complex situation is that the fibre-coating is operated in a cooling environment. In glass manufacturing process, glass fibres are made by drawing molten glass through an array of small diameter bushings. In order to enhance the heat removal from the fibres, they are sprayed with water from atomizing nozzles [10]. In this situation where the coating film is cooled by the environment, the Rayleigh-Plateau instability is modified by thermocapillary stress due to surface tension variations produced by temperature disturbances at the interface.

The effect of thermocapillarity on the dynamics of thin films on cylinders have given rise to broad scientific interest for its technological importance. Dávalos-Orozco and You [11] performed a linear stability analysis on the Navier-Stokes equations to investigate the three-dimensional thermocapillary instability of a fluid film coating the outside or the inside of a cylinder in the absence and in the presence of gravity. The results showed that pure thermocapillarity is possible to excite non-axisymmetric unstable mode. Liu and Liu [12] studied the longwave stability of thin film flowing down a uniformly heated vertical fibre. The results showed that the Marangoni instability and the Rayleigh-Plateau instability reinforce each other. With the increase of the thermocapillary effect, the coating flow has a tendency to break up into smaller droplets. Ding and Wong showed that these smaller droplets could also be unstable due to the azimuthal disturbances and would evolve into an asymmetric state [13]. Recently, Moctezuma-Sánchez, and Dávalos-Orozco [14] studied the non-axisymmetric longwave instability of a thin viscoelastic liquid film flowing down a vertical heated cylinder. The results show that, in comparison with the Newtonian case, it is easier to excite the azimuthal modes when viscoelasticity and thermocapillarity destabilize at the same time.

In experiments, the instability characteristics can be categorized by the location where instability growth can be visually detected. The concept of the convective/absolute stability was first developed in the context of plasma physics [15,16] and later has been extended to the problems of hydrodynamics [17]. Transitions between different wave regimes in coating flows on a fibre can be understood within the framework of absolute and convective instabilities. Convectively unstable flows behave as spatial amplifiers of the incoming perturbations: at a fixed point in the laboratory frame of reference, the signal eventually dies out. Whereas, absolutely unstable flows display intrinsic self-sustained dynamics: although advected, the perturbation is so strongly amplified that it contaminates the entire flow region (downstream and upstream).

Joo and Davis [18] have studied the absolute and convective stabilities for viscous falling films on a vertical plate. Recently, the absolute and convective instabilities of flows with a cylindrical free surface give rise to broad scientific interest. Duprat et al. [19] have studied the absolute and convective stabilities for a viscous film flowing down a vertical fibre. The authors have reported a flow regime diagram which identifies, depending on the fibre radius and the flow rate, the AI/CI characteristics. At large or small film thicknesses, the instability is convective, whereas absolute instability is observed in an intermediate range of film thicknesses for fibres of small enough radius. Balestra et al. [20] studied the linear spatio-temporal stability of heated coaxial jet flows. The results showed that the temperature ratio and the velocity ratio between the core jet play important roles in the transition from convectively to absolutely unstable flows.

In the present paper, we are interested in the aspect of the absolute and convective instabilities of a film flowing down a vertical fibre with a temperature difference between the fibre wall and the film interface.

This paper is organized as follows. In Section 2 the mathematical formulation of the physical model is presented. In Section 3 we present the results and discussions. In Section 4 we summarize the results and present the conclusions.

2. Mathematical formulation

As shown in Fig. 1, a Newtonian fluid, of constant viscosity μ and density ρ, flows down a vertical fibre of radius $r = a$ under gravity g. The initial radius of the fluid ring measured from the centre of the fibre is $r = R$. The temperatures of the fibre wall and the interface of the film are T_a and T_i.

The dynamics of the axisymmetric flow of the film is governed by the Navier-Stokes equations,

$$u_r + \frac{u}{r} + w_z = 0, \tag{1}$$

$$u_t + uu_r + wu_z = -\frac{p_r}{\rho} + \frac{\mu}{\rho}\left[u_{rr} + \frac{u_r}{r} - \frac{u}{r^2} + u_{zz}\right], \tag{2}$$

$$w_t + uw_r + ww_z = g - \frac{p_z}{\rho} + \frac{\mu}{\rho}\left[w_{rr} + \frac{w_r}{r} + w_{zz}\right], \tag{3}$$

$$T_t + uT_r + wT_z = \kappa\left[T_{rr} + \frac{T_r}{r} + T_{zz}\right], \tag{4}$$

where t denotes time, u and w denote the radial (r) and axial (z) velocity components, p denotes the pressure, T denotes the temperature, κ denotes the thermal diffusivity. Note that unless stated otherwise, the subscript denotes partial differentiation.

At the fibre surface ($r = a$), no penetration and no slip conditions for the velocities are

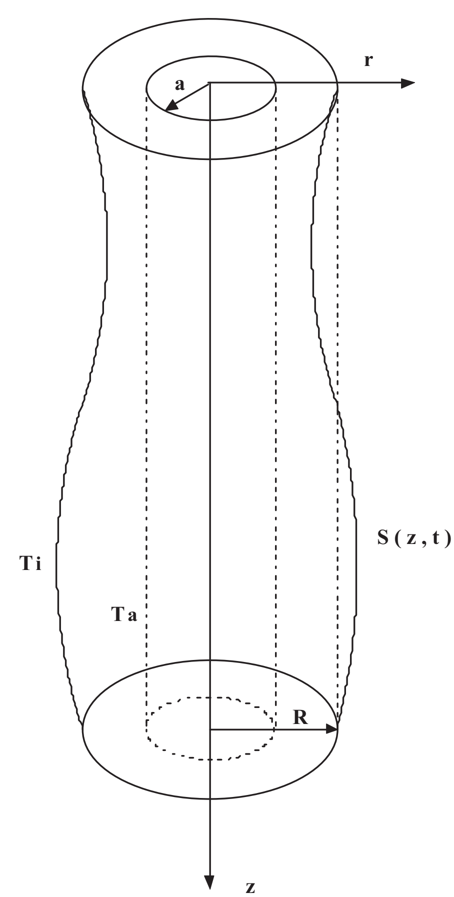

Fig. 1. Sketch of the geometry of a film flow coating a fibre.