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Solution of multi-dimensional radiative heat transfer in graded index media using the discrete transfer method

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ABSTRACT

The discrete transfer method is extended to solve the radiative transfer equation in two- and three-dimensional semi-transparent media with variable refractive index. The radiative transfer equation and other related equations are organized based on the refracted quantities. A cell-based ray tracing approach is used to track the curve paths of radiative rays in the medium. The complex geometries in multi-dimensional media are simulated by a simple blocked-off strategy. The present method is verified by comparing its results with benchmark solutions, and the performance of the method is examined by some numerical experiments.

1. Introduction

Radiative heat transfer in absorbing-emitting media plays an important role in many engineering applications, such as fuel combustion, rocket propulsion, hypersonic shock layers, and heat transfer in porous media. In all the cases mentioned above, the radiative rays propagate along straight lines. However, there are other engineering applications in semi-transparent media (STM) where the radiative rays travel along curved paths, due to spatial variation of refractive index. Such a STM with variable refractive index is known as graded index media (GIM). Glass manufacturing, thermal protection systems, solar systems, optical measurements of flames, and waveguide materials are some examples for engineering applications of GIM.

Radiative heat transfer in GIM is of great interest for many researchers in thermo-optical systems. Because of curve ray paths, the solution of radiative transfer equation (RTE) in GIM is more difficult than that in the media with constant refractive index. Many ray tracing techniques have been developed to solve the RTE in plane-parallel STM with variable refractive index [1–9]. Since the ray tracing is difficult in graded index media, other approaches have been developed to discretize the RTE in plane-parallel STM with spatial variation of refractive index [10,11].

Recently, radiative transfer in multi-dimensional GIM has paid more attention. Liu [12] proposed one of the earliest techniques, based on the finite volume method. Liu et al. [13–15] used finite element method to solve the RTE in multi-dimensional GIM. Benchmark solutions by the Monte Carlo method (MCM) for radiative transfer in two-dimensional GIM were presented by Liu [16]. Asllanaj and Fumeron [17] proposed a modified finite volume method to solve the RTE in two-dimensional GIM with irregular geometries. A hybrid finite volume-finite element method for solving RTE in complex geometries GIM was developed by Zhang et al. [18]. Other approaches have been developed based on the meshless method [19], spectral method [20,21], and so on. More recently, ray tracing approaches have been used for two-dimensional STM with variable refractive index [22,23].

The discrete transfer method (DTM) is known as a straightforward and intuitive method for solving the RTE in participating media [24,25]. Application of the DTM in solution of the RTE in plane-parallel GIM with linear variation of refractive index was developed by Krishna and Mishra [26], and improved by Hosseini Sarvari [27] to handle the plane-parallel GIM with arbitrary distribution of refractive index. The aim of this paper is to extend the application of the DTM for solving the RTE in multi-dimensional STM with spatial variation of refractive index. Two- or three-dimensional geometries, respectively, is divided into small-sized rectangular or cubic cells, and the hemisphere of solid angles is divided into elemental solid angles. Complex geometries are simulated by a simple blocked-off strategy. Then, the rays emanated from all surface elements are traced cell by cell along the curve paths. A simple and efficient cell-based approach is proposed for ray tracing. The solution method is verified by comparison the results with benchmark solution. Some numerical examples are presented to show the performance of solution method in semi transparent media with variable refractive index.

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Nomenclature

A	area, m^2
D	diameter, m
E_b	emissive power, W/m^2
F	blocked-off quantity
G	incident intensity, W/m^2 sr
H	length of the enclosure, m
I	radiative intensity, W/m^2 sr
n	refractive index
Q	radiative heat flux, W/m^2
s	geometric path length, m
T	temperature, K
V	volume, m^3
Greek symbols	
ε	emissivity
ϕ	azimuthal angle, rad
Γ	refracted incident intensity, W/m^2 sr
η	direction cosine
k	absorption coefficient, m^{-1}
θ	polar angle, rad
Θ	refracted heat flux, W/m^2
σ	Stefan-Boltzmann constant, W/m^2 K^4
τ	optical thickness
ω	weight
$\Im$	refracted intensity, W/m^2 sr
$\Re$	local refractive ratio
Subscripts	
b	blackbody
c	cell
w	wall
Superscripts	
i	incident direction
o	outward direction
r	reflected direction

2. Radiative transfer formulation

The RTE in an absorbing-emitting STM along the pencil of ray may be written as [27]:

$$\frac{d\Im}{ds} + \kappa\Im = \kappa I_b \quad (1)$$

where $\Im = I/n^2$ is known as the *refracted intensity*, and $I_b = \sigma T^4/\pi$ is the medium's blackbody intensity. Eq. (1) is a first-order differential equation which needs a boundary condition. For a diffuse-gray wall surface, the boundary condition is given by

$$\Im = \varepsilon_w I_{bw} + \frac{1-\varepsilon_w}{\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/2}\Im(\theta,\phi)\cos\theta\sin\theta d\theta d\phi \quad at\ s=0 \quad (2)$$

where the subscript w denotes the value at the wall surface. The *refracted radiative heat flux* through the physical domain, $\Theta = Q/n^2$, and over the wall surface, $\Theta_w = Q_w/n_w^2$, are calculated by

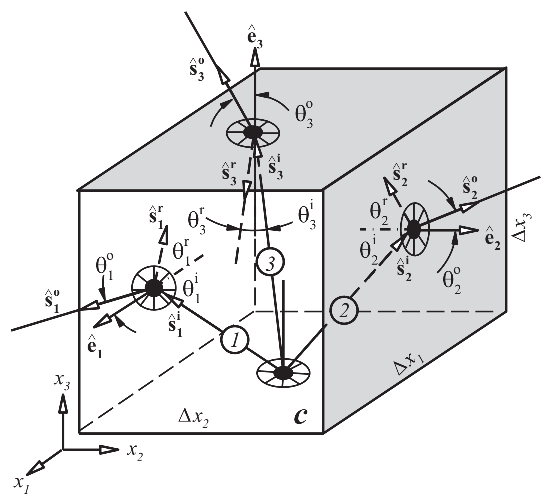

Fig. 2. Sample cubic cell and the rays propagated from the lower face.

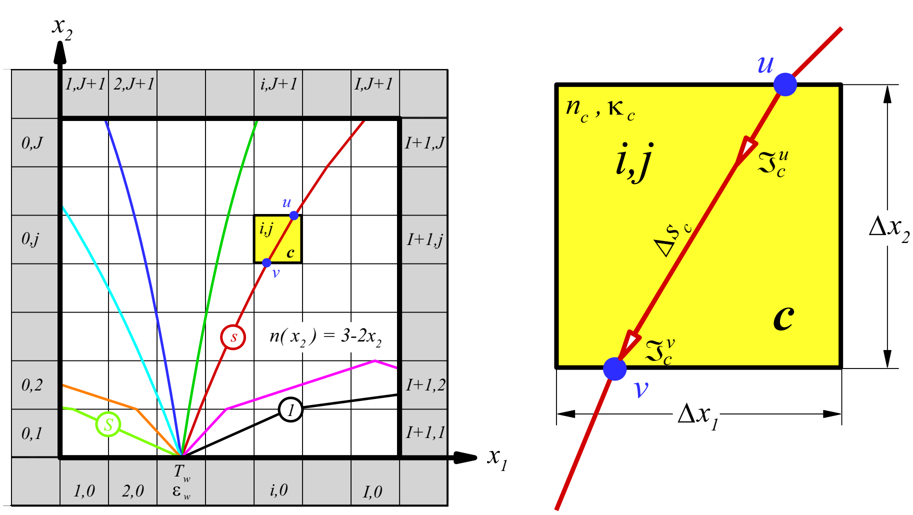

Fig. 1. (a) A schematic STM, spatial mesh, and ray trajectories, (b) a sample cell *c*.