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Turbulent Rayleigh-Bénard convection of cold water near its maximum density in a vertical cylindrical container

Xiao-Jie Huang, You-Rong Li *, Li Zhang, Chun-Mei Wu

Key Laboratory of Low-grade Energy Utilization Technologies and Systems of Ministry of Education, College of Power Engineering, Chongqing University, Chongqing 400044, China

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ABSTRACT

In order to understand the effects of density inversion parameter and Rayleigh number on the fluid motion and heat transfer characteristics of penetrative Rayleigh-Bénard convection in a cylindrical container, a series of large eddy simulations were performed. The working fluid is cold water and Prandtl number is 11.57. Rayleigh number up to 10^{11} is considered and the density inversion parameter ranges from 0 to 0.7. The results indicate that the effect of cold plumes on the larger scale circulation gradually diminishes and finally disappears with the increase of density inversion parameter. Furthermore, an increase of the temperature within the bulk region and a decrease of the penetration depth are also certified with the increase of density inversion parameter. With the increase of Rayleigh number, soft and hard turbulent states are successively observed. In the soft turbulence state, the evolution of Nusselt number shows a weak fluctuation, the corresponding histogram has a Gaussian distribution, and the penetration depth increases sharply with the increase of Rayleigh number. In the hard turbulence state, the evolution of Nusselt number presents a much stronger fluctuation than that in the soft turbulence state, the corresponding histogram fits more exponential distribution, and the variation of the penetration depth with Rayleigh number is small. A single power law between the heat transfer efficiency (Nusselt number) and Rayleigh number that covers soft and hard turbulence states is developed for each density inversion parameter. It is found that the average Nusselt number decreases linearly with the increase of density inversion parameter.

1. Introduction

Rayleigh-Bénard convection is a classical system of thermal convection, in which the fluid is heated from the bottom and cooled on the top by horizontal uniform boundary temperatures or heat fluxes [1–3]. Rayleigh-Bénard convection plays a crucial role in nature and many engineering fields, such as the convection in the atmosphere [4], in the earth [5], in the electronic devices and in the solar collectors and so on [6].

As early as 1987, Heslot et al. [7] and Castaing et al. [8] in the University of Chicago experimentally studied Rayleigh-Bénard convection of helium at low temperature. They observed a soft turbulence state ($2.5 \times 10^5 < Ra < 4 \times 10^7$) and a hard turbulence state ($4 \times 10^7 < Ra < 6 \times 10^{12}$). Then, many researchers paid their attention to soft and hard turbulence states in Rayleigh-Bénard convection by numerical and experimental methods. Peng et al. [9] analyzed the scaling relationship in an open-ended domain by Large Eddy Simulation (LES), where the Rayleigh (*Ra*) number varied from 6.3×10^5 to 10^9. They obtained a single relationship for the variation of Nusselt (*Nu*) number with Rayleigh number where the exponent is 0.286. Choi and Kim [10] numerically studied the scale of Rayleigh number with Nusselt number in the soft turbulence state ($2 \times 10^6 < Ra < 4 \times 10^7$) and the hard turbulence state ($10^8 < Ra < 10^9$) with the elliptic-blending second-moment closure. They predicted that Nusselt number follows two different correlations in soft and hard turbulence states. Shibata [11] calculated the heat conductivity in turbulent Rayleigh-Bénard convection. They found that the heat conductivity diverges stronger in the hard turbulence state than that in the soft turbulence state. Riedinger et al. [12] measured the heat flux in a slightly titled channel. They found that different flow regimes develop from a soft turbulence state to a hard turbulence state depending on the increase of the angle and the applied power. Qiu and Tong [13,14] experimentally studied the coherent events in an aspect-ratio-one cylindrical cell. A sharp transition from a random chaotic state to a correlated turbulence state is found when Rayleigh number exceeds 5×10^7, which offers new perceptions on the soft and hard turbulence states.

The above numerical simulation studies on Rayleigh-Bénard convection adopted the Oberbeck-Boussinesq approximation

* Corresponding author.
E-mail address: liyourong@cqu.edu.cn (Y.-R. Li).

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Nomenclature

C	coefficient in heat transfer correlation
c_w	subgrid turbulent model constant
f	oscillation frequency (Hz)
F	dimensionless oscillation frequency
g	gravity acceleration (m/s^2)
h_j	subgrid flux
H	height (m)
N	sampling points
Nu	Nusselt number
Nu_{ave}	average Nusselt number
Pr	Prandtl number
Pr_t	turbulent Prandtl number
$\overline{P}$	filtered pressure field
q	exponent in density-temperature equation
Ra	Rayleigh number based on the height
Ra_T	transient Rayleigh number
S_{ij}	the symmetric part of velocity gradient
T	temperature (K)
T_m	maximum density temperature (K)
U_0	reference velocity (m/s)
$\overline{U_i}$	filtered velocity component
W	width (m)
X_i	Cartesian coordinate (m)

Greek symbols

α	thermal diffusivity (m^2/s)
β	exponent in heat transfer correlation
γ	coefficient in density-temperature equation ($(°C)^{-q}$)
Δ	characteristic filter length (m)
δ_{iZ}	Kronecker delta
Θ	dimensionless temperature, $\Theta = (T - T_c)/(T_h - T_c)$
Θ_m	density inversion parameter
$\overline{\Theta}$	filtered temperature field
ρ	density (kg/m^3)
τ	dimensionless time
τ_{ij}	subgrid tensor
ν	kinematic viscosity (m^2/s)
ν_t	eddy viscosity (m^2/s)
Ω_{ij}	antisymmetric part of velocity gradient

[15,16], where the density of the fluid is assumed to be linear function of temperature. However, the density has the maximum value near 4 °C for the cold water, in which the Oberbeck-Boussinesq approximation is no longer applicable. This maximum density phenomenon changes the flow dynamics and heat transfer characteristics significantly. If the maximum density point is located between the bottom and top walls, the fluid in the convection cell could be divided into an unstable layer and a stratified stable layer. Convection occurs in the lower unstable fluid layer, the fluid motion penetrates to the upper stable layer. Therefore, it is called as penetrative convection. Veronis [17] employed the perturbation method to investigate how far the fluid motion penetrated into the stable layer and put forward the term "penetrative convection". Kuznetsova and Sibgatullin [18] described the evolution from conductive state to chaos in penetrative convection with the increase of Rayleigh number. A series of flow bifurcations were determined. Large and Andereck [19] carried out some experimental investigations on penetrative Rayleigh-Bénard convection in water near its maximum density point. The results showed that symmetric flow structures are formed if the whole layer is unstable. However, the symmetry will be destroyed by the existence of the stable layer. Mastiani et al. [20] and Hu et al. [21] investigated heat transfer characteristics of penetrative Rayleigh-Bénard convection. They found that heat transfer rate in penetrative Rayleigh-Bénard convection is lower than that in classical Rayleigh-Bénard convection.

The above existing literatures on penetrative convection are primarily concerned with laminar flow. Little attention has been paid to turbulent penetrative convection. Furthermore, there are a few investigations on turbulent penetrative convection in Rayleigh-Bénard system. In the present paper, we reported a series of the simulation results on turbulent Rayleigh-Bénard convection of cold water near its maximum density value by LES.

2. Problem formulation

2.1. Physical and mathematical model

We consider a cylindrical container with diameter D and height H. It is filled with cold water near its maximum density. The aspect ratio of the cylindrical container is $D/H = 1$. Constant temperatures T_c and T_h ($T_c < T_h$) are applied at the top and bottom walls, respectively. The sidewall is adiabatic. No-slip and impermeable conditions are imposed on all walls.

The density of cold water near 4 °C is described as follows [22],

$$\rho(T) = \rho_m(1 - \gamma|T - T_m|^q), \tag{1}$$

where $\gamma = 9.297173 \times 10^{-6}$ $(°C)^{-q}$, $q = 1.894\ 816$, $\rho_m = 999.972$ kg/m^3 and $T_m = 4.029\ 325$ °C. This density-temperature relation is applied for the buoyancy term in the momentum equation. Other physical parameters are assumed to be constant.

In this work, LES is used to simulate Rayleigh-Bénard convection of cold water. Using H, $U_0 = \sqrt{g\gamma(T_h - T_c)^q H}$, $\tau_0 = H/\sqrt{g\gamma(T_h - T_c)^q H}$ and $\rho_m(\sqrt{g\gamma(T_h - T_c)^q H})^2$ as the reference length, velocity, time and pressure, respectively. The dimensionless temperature is defined as $\Theta = (T - T_c)/(T_h - T_c)$. Then, the non-dimensional filtered equations are written as follows

$$\frac{\partial \overline{U_j}}{\partial X_j} = 0 \tag{2}$$

$$\frac{\partial \overline{U_i}}{\partial \tau} + \frac{\partial \overline{U_i}\,\overline{U_j}}{\partial X_j} = -\frac{\partial \overline{P}}{\partial X_i} + \left(\frac{Pr}{Ra}\right)^{1/2} \frac{\partial^2 \overline{U_i}}{\partial X_j \partial X_j} + \frac{\partial \tau_{ij}}{\partial X_j} + |\overline{\Theta} - \Theta_m|^q \delta_{iZ} \tag{3}$$

$$\frac{\partial \overline{\Theta}}{\partial \tau} + \frac{\partial \overline{\Theta U_j}}{\partial X_j} = (Pr \cdot Ra)^{-1/2} \frac{\partial^2 \overline{\Theta}}{\partial X_j \partial X_j} + \frac{\partial h_j}{\partial X_j} \tag{4}$$

where $Ra = g\gamma(T_h - T_c)^q H^3/(\nu\alpha)$ is the Rayleigh (Ra) number and $Pr = \nu/\alpha$ is the Prandtl (Pr) number. $\Theta_m = (T_m - T_c)/(T_h - T_c)$ is the density inversion parameter, which could describe the location of the maximum density temperature in regard to the bottom and top wall temperatures.

The key response of this system is the Nusselt (Nu) number, which is calculated as the surface average of the dimensionless heat transfer rate on the bottom wall, $Nu = -\langle \partial\Theta/\partial Z \rangle_A$, where $\langle \ldots \rangle_A$ stands for the average over the wall. In turbulent convection, in order to remove the effect of randomness, time average Nusselt number is introduced, which is denoted as $Nu_{ave} = -\langle \partial\Theta/\partial Z \rangle_{A,\tau}$, where $\langle \ldots \rangle_{A,\tau}$ denotes the average over the surface and the time.

In the LES process, scales which are smaller than the grid size are not resolved. In order to explain the subgrid turbulent effect