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Reflection and refraction of a thermal wave at an ideal interface

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ABSTRACT

Thermal waves are of great significance as non-Fourier effects arise with ultrafast heating rates and small system length. This study analytically and numerically investigated the behavior of thermal waves based on the Cattaneo-Vernotte model at an ideal interface. A stable, fast algorithm based on the alternative direction implicit method is introduced to solve the two-dimensional heat conduction problem. When thermal waves meet with an ideal interface, some energy is reflected back while the rest is conveyed across the interface, which are called the reflection and refraction of thermal waves. The changes of the profile and direction and the energy distribution between the reflection and refraction of the thermal waves are studied both analytically and numerically. Regardless of the boundary conditions imposed on the interface, the reflection angle is always identical to the incident angle, and the ratio of the sine of the refraction angle of the thermal waves to that of the incident angle is equal to the ratio of the thermal wave speeds in the two material layers. A theoretical equation to describe the relationships between the energy distribution and the material thermal properties shows that the thermal wave speeds in the materials, the specific heat and the incident angle determine the thermal energy transmittance ratio. Total reflection can occur for some conditions, and the nature of the energy conveyed by thermal waves is interesting and instructive.

1. Introduction

The thermal wave concept originates from the breakdown of the classical Fourier's law of heat conduction which implies an infinite propagation speed of a thermal perturbation [1,2]. Moreover, investigations of the second sound experimentally reveal that heat pulses propagate at finite speed with wave characteristics [3]. In particular, in nanomaterials pure diffusion predicted by Fourier's law fails to describe the heat conduction processes while lots of phonons are transported in the ballistic regime [4]. With the development of ultra-short pulse lasers and the miniaturization of electronic devices, thermal wave propagation in multilayered composite materials is of much significance as non-Fourier effects arise with these ultrafast heating rates and high heat fluxes. The wavelike behavior of thermal waves is widely recognized in biomedical sciences [5–8], microelectronics [9,10], nanomaterials [11–13], semiconductor materials [14–19] and other fields.

Several models have been proposed to characterize the propagation of thermal waves [20–27]. One of the thoughts is to consider that the diffusion coefficient is dependent on temperature and the thermal conductivity is zero in undisturbed medium to give a finite propagation speed [21]. Cattaneo-Vernotte model (CV) was proposed for the starting problem of gas based on kinetic theory [22,23]. Dual-phase-lag model (DPL) considers the relaxation terms for both heat flux and temperature gradient [24]. Phonon hydrodynamic model was proposed based on the solutions for the linear phonon Boltzmann equation [25,26]. Thermomass model (TM) describes the movement of thermomass based on the relation between energy and mass [27]. These models involve the inertia term, nonlocal term and nonlinear term, which could remove the paradox of the infinite heat perturbation speed in Fourier's law. However, the relaxation term of heat flux is thought to be the main factor to make it wavelike. In addition, several models would behave like CV model if given proper parameters. CV model is written as

$$q + \tau \frac{\partial q}{\partial t} = -k\nabla T, \tag{1}$$

where q is the heat flux, T is the temperature, k is the thermal conductivity and τ is the relaxation time. It is simple and typical to study the relaxation effect of thermal waves.

The propagation characteristics of thermal waves have been studied both experimentally and theoretically. Peshkov first measured the second sound velocity in helium II and obtained a speed of 19 m/s at 1.4 K [3]. More evidence of thermal waves was then

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Nomenclature

q	heat flux
x	x-coordinate of the domain
y	y-coordinate of the domain
L	length of the domain
t	time
t_0	duration time of imposed pulse
Δx	spatial steps
Δt	time steps
k	thermal conductivity
ρ	density
C_v	specific heat
A	amplitude of imposed pulse
Z	dimensionless relaxation time
T	temperature, period length
B	coefficient matrix
L	upper triangular matrix
U	lower triangular matrix
w	transitional vector
v	speed of thermal wave
r	thermal energy transmittance ratio
m	material speed ratio
G	integral of energy
ΔT_{theory}	temperature rise defined by heat flux
I_{heat}	thermal energy density
a	coefficient of Fourier decomposition
b	coefficient of Fourier decomposition

Greek symbols

τ	total energy
λ	coefficient of analytical solution
α	incident angle, thermal diffusivity
β	refraction angle
θ	reflection angle
ω	frequency of thermal waves
Δ	fluctuation of quantity

Superscripts

$*$	dimensionless parameter
n	time layer

Subscripts

0	reference state
1	materials in the left of interface
2	materials in the right of interface
ini	initial state
diff	diffusion process
ballistic	ballistic process
x	vectors in x-coordinate direction
y	vectors in y-coordinate direction
i	incident process
r	reflection process
t	refraction process
time	in time
space	in space
total	total energy
n	coefficient number
critical	critical value for total reflection
min	minimum value

found in other media, such as solid helium [28,29], NaF [30] and Bi [31]. Tsai and MacDonald [32] performed molecular dynamics (MD) simulations which showed thermal waves can be observed at room temperature or at even higher temperatures within tiny length and time scales. Lee et al. [12] performed first principle calculations and demonstrated that in hydrodynamic phonon transport processes, where R-scatterings can be neglected and momentum is conserved, the second sound phenomenon occurs. Yao et al. [11] used nonequilibrium MD simulations to study heat pulse propagation through graphene and found that thermal waves are transported in a ballistic way. Hua et al. [33] and Tang et al. [4,34,35] developed the phonon Monte Carlo (MC) methods for ballistic-diffusive thermal transport and studied ballistic thermal waves in a thin film.

As in light and sound, the wavelike characteristics of thermal waves are fascinating. The phenomena, such as overshooting [36,37], diffraction [38,39], reflection [40–47], refraction [48] and dispersion [49], have been investigated in detail. In particular, the behaviors of thermal waves at an interface have been studied for its importance in multilayered composite material heat conduction [5,9,50–59]. Tzou [48] used a harmonic analysis to study the reflection and refraction patterns of thermal waves from a surface and at an interface between dissimilar materials to show that the reflection angle depends on the ratio of the thermal wave speeds in both media. Bertolotti [15,40] used a mirage technique to experimentally demonstrate that Snell's law still fits the situation, indicating that the relationships between the angles and material properties still hold. Khadrawi et al. [60] studied the thermal behaviors of perfect and imperfect contact composite slabs using the hyperbolic heat conduction model. Lor and Chu [61,62] emphasized the significant influence of the thermal interface resistance, and extended numerical calculations to two-dimensional planes, in a study of reflected waves from insulated boundaries in a rectangular plate. Ho et al. [50] used the lattice Boltzmann method to investigate the heat transfer in multilayered materials within the framework of the dual-phase-lag (DPL) heat conduction model to show the temperature profiles after a heat pulse passed across an interface. Liu [63] analyzed metal films using the hyperbolic microscopic two-step model and found that the hyperbolic nature of heat in an electron gas significantly affects the thermal behaviors at early times. Al-Nimr et al. [9,53,54] investigated the DPL model for composite structures and considered the influence of the thermal boundary resistance, initial temperature, and material thermal properties on the thermal waves penetrating into another media.

Nevertheless, there are still few theories for the thermal energy distribution when heat pulses propagate through an interface. Previous studies have mainly focused on qualitative descriptions of the temperature profiles, with few discussions about the quantitative characteristics, which are important for precise thermal control in terms of thermal waves. In addition, the increases of system complexities require two-dimensional models to show how the heat propagates in complex structures and whether the principles drawn from one-dimensional assumptions are still applicable.

Interfacial thermal resistance, which is influenced by the properties of materials in contact, can largely affect the profiles of waves and energy transmittance ratio [54]. Hua and Cao [64,65] found that phonons in ballistic-diffusive regime generate boundary temperature jump because of the interactions between phonons