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Bayesian inferences of the thermal properties of a wall using temperature and heat flux measurements

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ABSTRACT

The assessment of the thermal properties of walls is essential for accurate building energy simulations that are needed to make effective energy-saving policies. These properties are usually investigated through in situ measurements of temperature and heat flux over extended time periods. The one-dimensional heat equation with unknown Dirichlet boundary conditions is used to model the heat transfer process through the wall. In Ruggeri et al. (2017), it was assessed the uncertainty about the thermal diffusivity parameter using different synthetic data sets. In this work, we adapt this methodology to an experimental study conducted in an environmental chamber, with measurements recorded every minute from temperature probes and heat flux sensors placed on both sides of a solid brick wall over a five-day period. The observed time series are locally averaged, according to a smoothing procedure determined by the solution of a criterion function optimization problem, to fit the required set of noise model assumptions. Therefore, after preprocessing, we can reasonably assume that the temperature and the heat flux measurements have stationary Gaussian noise and we can avoid working with full covariance matrices. The results show that our technique reduces the bias error of the estimated parameters when compared to other approaches. Finally, we compute the information gain under two experimental setups to recommend how the user can efficiently determine the duration of the measurement campaign and the range of the external temperature oscillation.

1. Introduction

Concerns about climate change and the effects of greenhouse gases have led to international targets for reducing carbon emissions [1,2]. One substantial source of carbon emissions is the built environment, which accounts for approximately one-third of global energy consumption [3]. For example, approximately 40% of national energy consumption in the UK is from the building sector. Reduction in carbon emissions from the built environment is, therefore, vital to meeting carbon reduction targets. Carbon emissions from buildings can be considerably reduced through large-scale policies that seek to limit energy demand for space heating and cooling [3]. Accurate predictions of building performance and energy demands are essential to the success of such policies. Specifically, computer simulations of heat loss from buildings are necessary to assess the effectiveness of energy-saving strategies such as retrofit interventions [4]. However, recent works [5–7] have shown that standard computer simulations of building performance may be unreliable due to inaccuracies from poorly characterized building structures including walls. Energy-saving measures based on inaccurate predictions of building performance may be economically ineffective.

Uncertainty in the thermal properties of walls is a primary source of inaccuracy in predictions of energy demand in buildings [7,8]. The heat capacitance and thermal conductance (resistance)

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Nomenclature

R	thermal resistance or R-value, m^2 K/W
ρ	density of the material, $\mathrm{kg/m}^3$
c_p	specific heat capacity, J/kg K
k	thermal conductivity, W/m K
ρC	heat capacity of unit area, $\mathrm{J/m}^2$ K
L	wall thickness, m
$T(x,t)$	temperature at position x and time t, $^\circ$C
T_0	initial temperature, $^\circ$C
$T_{int}(t), T_{ext}(t)$	surface temperatures of internal and external walls at time t, $^\circ$C
$\mathbf{T}_{int}, \mathbf{T}_{ext}$	surface temperatures of internal and external walls at times $t_0, \ldots, t_N$
$\mathbf{Y}_{int}, \mathbf{Y}_{ext}$	surface temperatures of internal and external walls at times $t_0, \ldots, t_N$
$\boldsymbol{\mu}_{int}, \boldsymbol{\mu}_{ext}$	smoothed surface temperature measurements of internal and external walls at times $t_0, \ldots, t_N$
C_{int}, C_{ext}	surface temperature noise covariance matrices of internal and external walls
$F_{int}(t), F_{ext}(t)$	heat fluxes of internal and external walls at time t, $\mathrm{W/m}^2$
$\mathbf{F}_{int}, \mathbf{F}_{ext}$	heat fluxes of internal and external walls at times $t_0, \ldots, t_N$
$\mathbf{Q}_{int}, \mathbf{Q}_{ext}$	heat flux measurements of internal and external walls at times $t_0, \ldots, t_N$
$\Sigma_{int}, \Sigma_{ext}$	heat flux noise covariance matrices of internal and external walls
θ	unknown parameters $(R, \rho C, \tau_0)$
λ	smoothing parameter
π	probability density function
π_p	prior probability density function
$\mathcal{L}$	likelihood function
ξ	experimental setup
D_{KL}	Information gain

of walls are used in standard heat transfer models as parameters for building performance simulations. Since these parameters of existing buildings are often unknown, the corresponding inputs in building simulations are typically obtained by visual inspection and tabulated values. In most cases, these values do not provide accurate characterizations of the walls of the building under consideration.

The thermal properties of walls can be inferred from in situ measurements of temperature and heat flux [9,10,7]. More specifically, the surface temperatures of internal and external walls denoted as $\{T^i_{int}\}^N_{i=1}$ and $\{T^i_{ext}\}^N_{i=1}$, are measured at a specified location over time. In addition, the heat flux through the wall, $\{q^i\}^N_{i=1}$, is also measured at N equispaced time points. ISO 9869:2014 [11] outlines a simple averaging procedure to determine the thermal transmittance (U-value) from in situ measurements. With this approach, the R-value (i.e., the inverse of the U-value) is computed directly by

$$R = \frac{\sum_{i=1}^N (T^i_{int} - T^i_{ext})}{\sum_{i=1}^N q^i}.$$

Since the averaging procedure assumes that the thermal mass of the wall is zero or almost zero, the accuracy of the estimate of the U-value will require measurements collected over an extended period of time (often longer than two weeks) [9,11]. More importantly, the averaging method does not provide a statistical framework that accounts for either the uncertainty in the thermal properties or errors in the measurements. As a result, this method fails to provide a proper quantification of the uncertainty in the estimated U-value of the wall.

Recent work has proposed the use of statistical approaches to infer thermal properties from in situ measurements of temperature and heat flux with simplified heat transfer models. In particular, a standard Bayesian inference has recently been proposed [9] to estimate the thermal properties of walls under the assumption that the heat dynamics of the wall can be described with a simple lumped-mass resistance–capacitance (RC) model. In contrast to the averaging method, the approach in [9] employs an RC network whose parameters include the thermal conductivity and the heat capacity. This standard Bayesian methodology suggests that these thermal properties can be inferred from in situ measurements based on relatively shorter measurement campaigns than the ones required by the averaging method. While other non-Bayesian statistical methods for estimating thermal properties have been proposed [12,9] provides substantial insight into the advantages of using Bayesian inference in building models and provides a motivation for further developments.

Our present work is a special application of parameter estimation for partial differential equations [13,14]. In particular, we develop and implement the hierarchical Bayesian approach introduced in [15]. Related works on Bayesian inference used for different applications can be found in the literature (see, for example, [16–18]). A general Bayesian formulation of inverse problems in heat transfer is also available in [19,20]. However, we address a problem where the boundary conditions can not be assumed known and a Bayesian approach based on the full likelihood function will not be recommended. Instead, the strength of our approach is to construct data-driven Gaussian priors [15], treating the boundary conditions as nuisance parameters to be marginalized out, to develop a quick and applicable Bayesian assessment of the parameters of interest. In [15], this approach was implemented to infer the thermal diffusivity parameter using synthetic temperature measurements in the interior and boundary of the domain. Here, we adapt the methodology to deal with temperature and heat flux measurements that are only available on the boundaries. We provide the maximum likelihood estimate (MLE) and the posterior distributions of the unknown parameters. Under the specification of independent uniform priors for the parameters of interest, we first use the Laplace method to produce fast estimates of their posterior distributions [21,22]. Then, we apply a Markov chain Monte Carlo (MCMC) sampling algorithm to assess the accuracy of the approximations obtained via Laplace method. The MCMC simulations, for this problem, support the employment of Laplace method to speed up the computations and estimate information gain values.

Most existing methodologies for inferring thermal properties [9] use forward models that can be derived from simplified coarse-grid approximations (often with 2 or 3 spatial nodes) of the heat equation that describes heat transfer through a wall. These simplified models are often used for the sake of computational expediency in the parameter identification process. However, such simplifications introduce intrinsic modeling errors that may, in turn, result in biased and potentially inaccurate estimated parameters. Alternatively, we use a heat equation with unknown Dirichlet boundary conditions to model the interior temperature of the wall, and we provide a convergence analysis to assess the effect of the discretization error in the Bayesian estimates of the