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# Effective permeability of fractal fracture rocks: Significance of turbulent flow and fractal scaling

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ABSTRACT

In this study, we develop a new approach to calculate hydraulic gradient dependent effective permeability of a fractal fracture network where both laminar and turbulent flows may occur in individual fractures. The cubic law is used to calculate flow behaviors in fractures where flow is laminar, while the Forchheimer's law is used to quantify turbulent flow behaviors. A critical fracture length is used to distinguish flow characteristics in individual fractures. While flows in some fractures may be turbulent, we assume that the fractal fracture network can still be treated as a porous medium where Darcy's law applies and our objective is to determine an effective permeability for the network. The developed new solutions can also be used for the case of a general scaling relationship, an extension to the linear scaling. We examine the impact of fractal fracture network characteristics on the effective permeability of the network. These characteristics include: fractal scaling coefficient and exponent, fractal dimension, ratio of minimum over maximum fracture lengths. The influence of imposed hydraulic gradient and critical length on the effective permeability is also examined and discussed. Results demonstrated that the developed solution can explain more variations of the effective permeability in relation to the fractal dimensions estimated from field observations. At high hydraulic gradient the effective permeability decreases with the fractal scaling exponent, but increases with the fractal scaling exponent at low gradient. The effective permeability increases with the scaling coefficient, fractal dimension, fracture length ratio and maximum fracture length.



## 1. Introduction

Fluid flow in fracture rock is mostly controlled by networks of interconnected conductive fractures. Several methods for modeling flow in fracture media have been developed in the past. Discrete fracture network (DFN) and stochastic continuum (SC) are two common approaches used for simulating fluid flow and solute transport in fracture media [1]. The DFN approach requires information about the hydraulic properties of individual fractures in the network [2–5], where flow is then calculated in each fracture individually. Because the permeability of fracture networks is usually much greater than that of the rock matrix in the fracture rock masses, the DFN approach that neglects fluid flow in rock matrix has gained much attention recently. In the SC approach, the fracture rock is represented as an equivalent homogeneous porous medium with the flow being governed by Darcy's law. The permeability of the equivalent medium is treated as the effective permeability that represents the spatially averaged fracture properties [6–8]. Liu et al. [9] recently provided a review on studies of estimating equivalent permeability of two dimensional (2-D) DFNs considering the influences of geometric properties of fracture rock masses. Mathematical expressions for the effects of a list of important parameters that significantly impact on the effective permeability of DFNs were summarized and discussed.

Some recent experimental observations demonstrated the importance of turbulent flows in fractures [10,11]. Many studies presumed that fluid flow in each single fracture is laminar and follows the cubic law [12,13]. However, the cubic law that neglects the inertial effects can only be valid for sufficiently low Reynolds numbers ($R_e$). Previous studies have demonstrated that the flow rate could be nonlinearly related to the hydraulic gradient when the applied pressure/flowrate is large [14–16]. Studies on fluid flow through crossed fractures revealed complex nonlinear flow patterns when $1 < R_e < 100$ [17,18]. The $R_e$ has been typically incorporated to determine the nonlinear flow through single fractures, which may not apply to DFNs, because flow in each single fracture can have a different $R_e$. The assessment of the values of localized $R_e$ in the DFNs with large number of fractures would be a tough challenge [11].

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Fractures in rocks have been shown to have self-similar and fractal feature [19,20]. Yu and Cheng [21] showed that fractal model can reveal more mechanisms of fluid flow in porous media than traditional model. Yun et al. [22] examined the starting pressure gradient and permeability for non-Newtonian flow characteristics in fractal porous media. Xu et al. [23] developed analytical expressions for the thermal conductivity of fractal-like tree networks. The idea of fractal porous media has also been extended to unsaturated seepage problems [24]. Most recently, Jin et al. [25] used four fractal dimensions for the development of a mathematical model to describe fluid flow in fractal tight porous media. A unified model connecting the porosity and the fractal dimension was established for fractal porous media, in which the Hagen-Poiseuille equation for laminar flow and the fractal assumptions were used to derive a permeability model for fractal media. Miao et al. [26] used the fractal geometry theory to investigate seepage characteristics and develop analytical expressions for the effective permeability of fractal fracture networks, which included microstructure parameters of fractures and no other empirical constants. Liu et al. [27] studied the permeability of fractal fracture network, in which the fracture length followed a power law distribution using the Monte Carlo method. Jin et al. [28] established a triple-effect permeability model of rough fractures according to the functioning patterns of surface tortuous effect, hydraulic tortuous effect and local stationary surface roughness factor and reformulated it into a scaling form for a self-affine fracture. One of the main features of fractal fracture network is that the relation between the fracture length ($l$) and the opening displacement (aperture) of the fracture populations ($a$) can be expressed as $a = \beta l^n$, where $\beta$ is a scaling coefficient and $n$ a scaling exponent [29]. At 90% confidence level, Hatton et al. [29] found a distinct change in scaling from $n \approx 2$ to $n \approx 1$ at fracture length of ~10 times the mechanical grain size provided by the cooling joints.

To summarize, previous studies have mostly dealt with effective permeability of fractal fracture network in which flows in all fractures in the network were typically assumed to be laminar and the relationship between the aperture and length followed a linear scaling relationship. Our main objectives in this study are to examine how turbulent flow and fractal scaling may affect the effective permeability of fractal fracture network. We develop new solutions applicable to the fractal fracture network where laminar flow and turbulent flow may occur in different individual fractures. We incorporate a critical fracture aperture in distinguishing flow behavior in each individual fracture because $R_e$ is linearly related to the aperture. Since the aperture is proportional to the length according to fractal scaling, critical $R_e$ can be approximately related to a critical length $l_c$. In fracture with length greater than $l_c$, the flow can be considered as turbulent and described by the Forchheimer's law. In other fractures where flows are laminar, we use the widely used cubic law. Therefore, the transition between laminar and turbulent flows occurs at a critical fracture length. Furthermore, the developed new solutions of effective permeability can also be used for the case of a general scaling relationship, an extension to the linear scaling. In particular, we treated $n$ as a variable and examined the impact of $n$ on the effective permeability, unlike the study of Miao et al. [26] where the value of $n = 1$ was used indicating a linear scaling law. While flows in some fractures may be turbulent, we assume as a whole the fractal fracture network can still be treated as a porous medium where Darcy's law applies and our task is to determine an effective permeability for the fractal fracture network, which is hydraulic gradient dependent.

## 2. Methods

### 2.1. Fractal fracture network characteristics

The width between plates/walls of a fracture, i.e., the parallel plate model is used to represent the effective aperture of a fracture. Generally, the relationship between the effective aperture $a$ and the fracture trace length $l$ is given by [29,30],

$$a = \beta l^n \tag{1}$$

where $\beta$ and $n$ are the proportionality coefficient and the scaling exponent according to fracture scales, respectively. The value of $n = 1$ is a special case indicating a linear scaling law and the facture network is self-similar and fractal [26,30,31].

Similar to the cumulative size distribution of islands on the Earth surface [32], the cumulative distribution of fracture area obeys the fractal law,

$$N(S \geqslant s) = \left(\frac{a_{max} l_{max}}{al}\right)^{D_f/2} \tag{2}$$

where $N$ is the total number of fractures with area greater than $s = al$, $a_{max} l_{max}$ represents the maximum fracture area with $a_{max}$ and $l_{max}$ being the maximum aperture and maximum fracture length, respectively.

Therefore, we can obtain the following relation by substituting Eq. (1) into Eq. (2),

$$N(L \geqslant l) = \left(\frac{\beta l_{max}^{n+1}}{\beta l^{n+1}}\right)^{D_f/2} = \left(\frac{l_{max}}{l}\right)^{\frac{(n+1)D_f}{2}} \tag{3}$$

where $D_f$ is the fractal dimension.

The number of fractures that is in the range from $l$ to $l + dl$ is then,

$$-dN(l) = \frac{(n+1)D_f}{2} l_{max}^{\frac{(n+1)D_f}{2}} l^{-\frac{1}{2}[(n+1)D_f+2]} dl \tag{4}$$

Miao et al. [26] extended the concept of fractal dimension $D_f$ in relation to the porosity and pore size in a porous medium [33] as,

$$D_f = d_E + \frac{ln\phi}{ln(l_{max}/l_{min})} \tag{5}$$

where $\phi$ is the effective porosity of fractal fracture network in a rock. From the above equation, we can relate porosity $\phi$ to the minimum and maximum length ratio of a two-dimensional fractal fracture network ($d_E = 2$) as follows,

$$\phi = \left(\frac{l_{min}}{l_{max}}\right)^{(2-D_f)} \tag{6}$$

The effective porosity can also be expressed by definition as,

$$\phi = \frac{A_f}{A} \tag{7}$$

where $A$ is the area that is represented by the effective permeability, and $A_f$ is the total area of all fractures in the area, which can be obtained from the following integration,

$$A_f = -\int_{l_{min}}^{l_{max}} a l dN(l) = \frac{\beta(n+1)D_f}{2+2n-(n+1)D_f} l_{max}^{(1+n)} \left\{1 - \left(\frac{l_{min}}{l_{max}}\right)^{\frac{1}{2}[2+2n-(n+1)D_f]}\right\} \tag{8}$$

Then the area $A$ can be related to the total facture area through porosity as follows,