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A new hydrodynamic boundary condition simulating the effect of rough boundaries on the onset of Rayleigh-Bénard convection

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ABSTRACT

This paper introduces a new hydrodynamic boundary condition which enables the simulation of the effects caused by rough boundaries. The classical Rayleigh-Bénard stability analysis is performed here to investigate the onset of thermal convection in a parallel-plate channel with rough boundaries. The hydrodynamic boundary conditions are modified, from the classical treatment, in order to consider channel boundaries characterised by non-negligible roughness. This roughness is simulated as a shallow fluid saturated porous medium and the Saffman interface condition is thus employed to model the hydrodynamic boundary conditions. The normal mode method is employed and the obtained eigenvalue problem is solved numerically. The Principle of Exchange of Stabilities is proved and the critical values of the Rayleigh number and of the wave number are obtained.

1. Introduction

The effect of boundary roughness on turbulent flows is well studied, see Jimenez [1]. Boundary roughness is important when transition from laminar to turbulent flow in pipes is investigated, because for smooth boundaries the linear stability analysis predicts infinitely stable flow, Cotrell [2]. Roughness of the porous/fluid interface is also shown to have significant effects on turbulent flows in composite porous/fluid domains, even if turbulence in the porous domain is neglected, Kuznetsov [3]. In laminar flows, which are stratified, the effect of boundary roughness is usually negligible. The situation may be different when flows in microchannels are considered. Nield and Kuznetsov [4] have shown that laminar flows in such channels may be very sensitive to any boundary roughness because in microchannels boundary roughness could occupy a significant portion of the channel width. Boundary roughness may also be important in narrow biological cavities, such as the nodal pit, Kuznetsov [5].

The cellular flow field generated by temperature gradient induced density variations is called Rayleigh-Bénard convection [6] and is one of the best known problems in natural convection. In such cases, buoyancy is the main mechanism responsible for triggering the instability that drives the flow. In this paper we investigate the effect of large boundary roughness on the onset of Rayleigh-Bénard convection in a parallel-plate channel with rough boundaries. Because the exact profile of the rough surface could be very complex, modeling this problem requires suggesting a new boundary condition that would simulate hydrodynamic phenomena at the rough boundary. In order to account for the additional momentum loss due to the rough portion of the boundary we propose simulating this region by replacing it with a thin porous layer. The problem then becomes similar to a convection problem in a composite domain consisting of a clear fluid core and two porous regions at the boundaries, see, for example, Kuznetsov [7].

2. Mathematical model

A Newtonian fluid saturating a horizontal channel is studied here to establish the onset of buoyancy driven convective instability. The two horizontal plates that bound the channel are impermeable and separated by a distance H. The classical Rayleigh-Bénard configuration is assumed: the lower boundary is held at a temperature $T_0 + \Delta T$ and the upper boundary is held at a temperature T_0. On assuming the Oberbeck-Boussinesq approximation, the conservation equations that describe the system can be written as follows

$$\overline{\nabla} \cdot \overline{\mathbf{u}} = 0,$$
$$\rho \frac{\partial \overline{\mathbf{u}}}{\partial \overline{t}} + \rho \, \overline{\mathbf{u}} \cdot \overline{\nabla} \overline{\mathbf{u}} = -\overline{\nabla} \overline{p} - g \beta (\overline{T} - T_0) \mathbf{e}_z + \mu \overline{\nabla}^2 \overline{\mathbf{u}},$$
$$\frac{\partial \overline{T}}{\partial \overline{t}} + \overline{\mathbf{u}} \cdot \overline{\nabla} \overline{T} = \alpha \overline{\nabla}^2 \overline{T}, \qquad (1)$$

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Nomenclature

$\mathbf{e}_z$	unit vector in the $z-$direction
f	eigenfunction, Eq. (9)
g	gravity
h	eigenfunction, Eq. (9)
H	channel height
k	wavenumber, Eq. (9)
(K_1, K_2)	permeabilities of the shallow porous boundaries
p	pressure
P	pressure disturbance, Eq. (7)
Pr	Prandtl number, Eq. (4)
Ra	Rayleigh number, Eq. (4)
t	time
T_0	dimensional reference temperature
T	temperature
$\mathbf{u}$	velocity vector, (u, v, w)
$\mathbf{U}$	velocity disturbance vector, (U, V, W), Eq. (7)
$\mathbf{x}$	position vector, (x, y, z)

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
ΔT	temperature gap between the boundaries
ϵ	dimensionless perturbation parameter, Eq. (7)
η	perturbation growth rate, Eq. (9)
Θ	temperature disturbance, Eq. (7)
$\Lambda_1,\ \Lambda_2$	dimensionless parameters, Eq. (4)
μ	dynamic viscosity
(ξ_1, ξ_2)	dimensionless parameters, Eq. (2)
ρ	density
(χ_1, χ_2)	initial conditions employed in the shooting method, Eq. (17)
Ψ	streamfunction
ω	perturbation angular frequency, Eq. (9)

Superscript, subscripts

$\bar{}$	dimensional quantity
$\star$	complex conjugate
b	basic state
c	critical value
$'$	differentiation with respect to z

where $\mathbf{u} = (u, v, w)$ is the velocity vector, ρ is the fluid density, t is the time, g is the gravity, β is the thermal expansion coefficient of the fluid, $\mathbf{e}_z$ is the unit vector in the $z-$direction, p is the excess pressure (above hydrostatic), μ is the hydrodynamic viscosity of the fluid, T is the temperature and α is the thermal diffusivity. The thermophysical properties of the fluid are evaluated at a reference temperature T_0.

The roughness of the channel boundaries is considered non-negligible, see Fig. 1, such that the two boundaries cannot be treated as relatively smooth horizontal surfaces. The expression "relatively smooth surface" means that the roughness of the surface is negligible but the surface is not perfectly smooth in the sense of Cotrell [2]. The roughness is a white noise space oscillation in the z-direction, where the average value identifies the position of the boundary. The aim of this study is to investigate how this roughness may influence the onset of Rayleigh-Bénard instability. The presence of the roughness is modelled by assuming that the two horizontal boundaries are shallow porous layers. This assumption allows one to employ, as a hydrodynamic boundary condition, the Saffman interface condition [8] between a clear fluid and a fluid saturated porous medium. The vertical velocity at the boundaries is set equal to zero as a consequence of the assumption of shallow porous layers. The hydrodynamic and thermal boundary conditions may thus be expressed as follows

$$\begin{aligned} &\bar{z}=0: \quad \frac{\partial \bar{u}}{\partial \bar{z}} - \frac{\xi_1}{\sqrt{K_1}}\bar{u} = 0, \quad \frac{\partial \bar{v}}{\partial \bar{z}} - \frac{\xi_1}{\sqrt{K_1}}\bar{v} = 0, \quad \bar{w} = 0, \quad \bar{T} = T_0 + \Delta T, \\ &\bar{z}=H: \quad \frac{\partial \bar{u}}{\partial \bar{z}} + \frac{\xi_2}{\sqrt{K_2}}\bar{u} = 0, \quad \frac{\partial \bar{v}}{\partial \bar{z}} + \frac{\xi_2}{\sqrt{K_2}}\bar{v} = 0, \quad \bar{w} = 0, \quad \bar{T} = T_0, \end{aligned} \tag{2}$$

where ξ_1 and ξ_2 are dimensionless parameters that strongly depend on the pore size of the shallow porous medium and thus on the geometry of the interface between the clear fluid and the shallow porous medium. Beavers and Joseph [9] have shown experimentally that the parameter ξ can be correlated directly with the average pore diameter at the interface. On the other hand, K_1 and K_2 are the permeabilities of the two shallow porous regions. A low value of K_i implies that the porous medium behaves as a solid material. In this case, a relatively smooth surface is obtained and the no-slip conditions are recovered: if one takes the limit $K_i \to 0$, the boundary conditions at both $\bar{z} = 0$ and $\bar{z} = H$ indeed become $\bar{u} = \bar{v} = 0$, Eq. (2). When a high value of permeability is assumed, a vanishing porous matrix is obtained such that a zero excess pressure boundary condition at the boundary is obtained: if one takes the limit $K_i \to \infty$, the boundary conditions at both $\bar{z} = 0$ and $\bar{z} = H$ become $\partial \bar{u}/\partial \bar{z} = \partial \bar{v}/\partial \bar{z} = 0$, Eq. (2). The governing equations and boundary conditions can now be written in a dimensionless form, namely

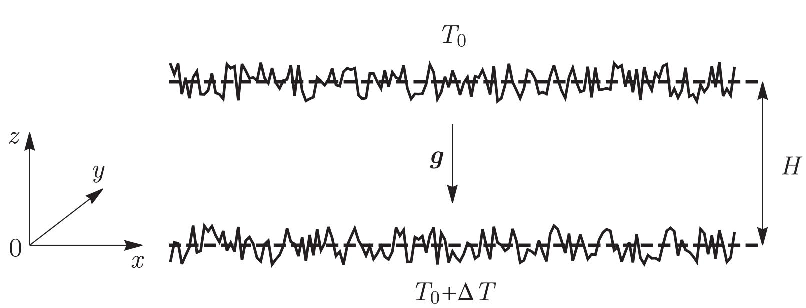

Fig. 1. Channel boundaries roughness and thermal boundary conditions.