



# Inverse analysis of laser surface heating from thermal strain measurements



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## ABSTRACT

This study proposes an inverse heat conduction methodology for laser heating treatment of a thin cylinder bonded to a strain gauge. The shifting function method is applied twice to respectively generate analytical solutions of temperature and thermal strain functions by giving the general type of unknown temperature function at the heated end. Afterwards we used the least-squares method to minimize the difference between theoretical calculated strains and the measured strains at an interior location at the discrete measurement times; therefore, the whole temperature and heat flux functions of laser heating process can be directly generated. The proposed methodology benefits the researches by avoiding complex numerical operations, reducing the rank of the coefficient matrix of the least-squares method from five to three, and setting the strain gauge on the cylinder surface easily. At last, one mathematical and one experimental examples are used to demonstrate the accuracy and efficiency of this work via using thermal strain measurements.

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## 1. Introduction

The inverse heat conduction problem (IHCP) is usually defined as the estimation of the surface heat flux and/or temperature histories given one or more measured temperature histories inside a heat-conducting body and has been written in textbooks [1–3]. Typical examples consist of heat exchangers, combustion chambers, calorimeter-type instrument, and a shuttle or missile reentering the earth's atmosphere from space. Besides, the laser surface treatment of an object, including surface hardening, cladding, and plating, has been widely used in the past decades in industrial process. In general, its surface temperature must be held above the critical transformation temperature, but less than the melting point of the object in the surface hardening process. Therefore, the IHCP of laser heat treatment is important in engineering applications. Because an IHCP problem is attributed to an ill-posed mathematical problem, it requires both the inverse analysis schemes and the extra experimental measurements. We will discuss them below in detail.

Regarding the inverse analysis schemes, many numerical techniques use finite difference method, finite element method, and/or Laplace transform method to solve the IHCP. For the problem of laser heat treatment on a surface of a cylinder [4–7], Wang et al. [5] in 2000 conducted an experiment to measure interior

temperatures using the thermocouples and performed an inverse study by the conjugate gradient method. They found that the sensor location nearby the heated surface can obtain more accurate surface temperature. Later, Chen and Wu [6] proposed a hybrid technique of Laplace transform and finite difference method to estimate the laser heated surface temperature by using the experimental data of Ref. [5]. Generally speaking, common methods of IHCP must tackle stability in numerical schemes, use large numbers of cells or elements in matrix operation, and perform the complex inverse Laplace transform, resulting in inefficient and redundant efforts. In 2014, Lee and Huang [7] developed an integral-transform-free methodology for one-dimensional IHCP with time-dependent boundary conditions to estimate the heat flux on the heated surface of laser heat treatment. Because the heating time is short in laser heat process, they approximated the unknown surface temperature using a fourth-degree polynomial function (five undetermined coefficients) of time and utilized the shifting function method to generate a closed solution. The unknown coefficients of the polynomial function could be determined by using the least-squares method together with the analytical solution and temperature measurements. In a companion study, they [8] divided the whole time domain into several sub-time intervals to deal with long-time spray cooling problems. Recently, without dividing the time domain, Lee and Yan [9] proposed the half-range Fourier cosine to expand the unknown time-dependent temperature function on the entire time domain.

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## Nomenclature

### Scalar

$B_{ni}(i = 1 \sim 6)$	auxiliary functions defined in Eqs. (B10)–(B15)
$\bar{B}_{ni}(i = 1 \sim 6)$	dimensionless auxiliary functions used to simplify equations
$c$	specific heat ( $\text{W} \cdot \text{s}/\text{kg} \cdot ^\circ\text{C}$ )
$c_0$	parameter defined in the second term of Eq. (8)
$d_i(i = 1 \sim 4)$	constants used to express time-dependent temperature function
$D$	the diameter of the test cylinder (m)
$D_i(i = 1 \sim 4)$	elements of matrix $\mathbf{D}$ and equal to $d_i(i = 1 \sim 4)$
$e$	error of a strain gauge
$E$	Young's modulus
$\bar{E}$	error function
$f$	real time-dependent temperature function at the heated end
$\tilde{f}$	time-dependent temperature function minus $T_0$ at the heated end
$\bar{f}$	dimensionless quantity of $\tilde{f}$
$g_T, g_u$	shifting functions for temperature and displacement functions
$G$	shear modulus ( $\text{N}/\text{m}^2$ )
$k$	thermal conductivity ( $\text{W}/\text{m} \cdot ^\circ\text{C}$ )
$k_0$	parameter defined in the first term of Eq. (8)
$\bar{K}$	dimensionless quantity of $k_0 T_0$
$L$	the length of the test cylinder (m)
$N_n$	norm of trial functions
$p$	number of measured times
$\bar{p}$	dimensionless quantity defined below Eq. (66)
$q$	heat flux function
$q_{Tn}, q_{un}$	time-dependent generalized coordinate for temperature and displacement functions
$\bar{q}_{Tn}, \bar{q}_{un}$	dimensionless quantities of $q_{Tn}, q_{un}$
$r$	radial coordinate (m)
$R_i$	elements of matrix $\mathbf{R}$
$t$	time variable (sec)
$t_m$	terminated time of laser beam application
$t_r$	discrete measured time (sec)
$T$	real temperature function ( $^\circ\text{C}$ )
$\bar{T}$	equal to $T$ minus $T_0$ ( $^\circ\text{C}$ )
$T_0$	constant initial surrounding temperature ( $^\circ\text{C}$ )
$u, u_r, u_\theta$	displacement functions in $x-, r-, \theta-$ directions
$U, V$	transformed functions for temperature and displacement functions

$U_0$	initial value of transformed function $U$
$x$	space coordinate (m)
$x_m$	measurement position of strain gauge (m)
$X, X_m$	dimensionless quantities of $x$ and $x_m$
$Z_{ij}$	elements of matrix $\mathbf{Z}$

### Matrix and vector

$\mathbf{D}$	vector defined in Eq. (60)
$\mathbf{R}$	vector defined in Eq. (60)
$\mathbf{Z}$	coefficient matrix defined in Eq. (60)

### Greek symbols

$\alpha$	thermal diffusivity ( $\text{m}^2/\text{s}$ )
$\alpha_t$	linear coefficient of thermal expansion
$\beta$	dimensionless quantity of $u$
$\delta_i(i = 1 \sim 4)$	quantities defined in Eq. (B4)
$\bar{\delta}_i(i = 1 \sim 4)$	dimensionless quantities defined in Eq. (C15)
$\delta_{ir}$	the abbreviation of $\delta_i(x_m, t_r)$
$\varepsilon, \varepsilon_r, \varepsilon_\theta$	thermal strain functions in $x-, r-, \theta-$ directions
$\varepsilon^{meas}, \varepsilon^{exact}$	measured and exact thermal strains
$\phi_{Tn}, \phi_{un}$	eigenfunctions for temperature and displacement functions
$\varphi$	auxiliary integration variable
$\gamma_{Tn}, \gamma_{un}$	auxiliary functions for temperature and displacement functions
$\eta_l(l = 1 \sim 4)$	quantities defined in Eq. (57)
$\bar{\eta}_l(l = 1 \sim 4)$	dimensionless quantities defined in Eqs. (C21)–(C24)
$\eta_{lr}$	the abbreviation of $\eta_l(x_m, t_r)$
$\lambda_n$	eigenvalue
$\mu$	standard deviation
$\nu$	Poisson's ratio
$\theta$	circumference coordinate
$\bar{\theta}, \bar{\theta}_0$	dimensionless temperature functions
$\rho$	mass density ( $\text{kg}/\text{m}^3$ )
$\sigma, \sigma_r, \sigma_\theta$	thermal stresses ( $\text{N}/\text{m}^2$ )
$\tau, \tau_{mA}, \tau_{mB}, \tau_i(i = 1 \sim 4)$	Fourier numbers
$\xi_n$	auxiliary relationship

### Subscripts

$O, A, B, i, j, k, l, m, n, r, t, T, u, \theta -$

Regarding the experimental methods utilizing IHCP, thermocouple and strain gauge is two different choices, taking temperature and thermal strain at the interior location, respectively. To date, most of work in IHCP has been limited to use the temperature measurements [1–9]; however, the thermocouple might not be the most appropriate sensor to take the internal measurements. In the literature, only a few investigations [10–12] predicted the unknown surface condition via using the strain gauge sensors. In 1981, Gyrsa et al. [10] combined the thermal stresses theory with the Laplace transform method to investigate the temperature field from the temperature, heat flux and displacement measurements inside the solid; even so, the inversion of the unknown surface temperature in the transform domain was cumbersome, resulting in the difficulty of inverting the unknown surface temperature in the transform domain to the physical quantity. Although they suggested that the sensor must be located near the position of the unknown boundary condition to obtain a more accurate estimated result, their estimated results were sensitive to the internal mea-

surements and the magnitude of the time-step. Later, Blanc and Raynaud [11] developed an analysis along with the quasi-static and uncoupled assumptions to obtain the unknown boundary condition of an IHCP by using the thermal strain and temperature measurements instead of the temperature measurements only. The discrete form of Duhamel's integral and future time steps was presented by Taler and Zborowski [12] to study IHCP concerning thermal stress control in elements of complex shapes. In 2006, Chen et al. [13] applied a hybrid numerical algorithm of the Laplace transform technique, the finite-difference method with a sequential-in-time concept, and the least-squares scheme to predict the unknown surface condition from the theory of dynamic thermal stresses. As far as we know, none of the literature performed the estimation of heat flux of laser heat treatment by using the strain measurements.

This work develops a strain gauge measurement method to measure the thermal strain and performs the inverse analysis of laser heating process for the first time. We temporarily gave the

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