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Review

Entrance-length dendritic plate heat exchangers

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ABSTRACT

Here we explore the idea that the highest heat transfer rate between two fluids in a given volume is achieved when plate channel lengths are given by the thermal entrance length, i.e., when the thermal boundary layers meet at the exit of each channel. The overall design can be thought of an elemental construct of a dendritic heat exchanger, which consists of two tree-shaped streams arranged in cross flow. Every channel is as long as the thermal entrance length of the developing flow that resides in that channel. The results indicate that the overall design will change with the total volume and total number of channels. We found that the lengths of the surfaces swept in cross flow would have to decrease sizably as number of channels increases, while exhibiting mild decreases as total volume increases. The aspect ratio of each surface swept by fluid in cross flow should be approximately square, independent of total number of channels and volume. We also found that the minimum pumping power decreases sensibly as the total number of channels and the volume increase. The maximized heat transfer rate per unit volume increases sharply as the total volume decreases, in agreement with the natural evolution toward miniaturization in technology.

Contents

1. Technology evolution

Technology evolution [1,2] is emerging as the most common manifestation of the universal tendency toward evolutionary design in nature, which in physics is summarized as the constructal law. Technology evolves visibly in our lifetime, and reveals the physics meaning of the evolution phenomenon: evolution means changes that occur freely in a flow architecture over time, in a discernable direction in time from the point of view of the observer. The movie tape of evolutionary images runs in one direction, forward. Think of evolution as the phenomenon of geometric irreversibility.

The physics of evolution had its start in engineering [2,3]. Since 1996, this physics phenomenon and law have guided theoretical and applied investigations that feed an active field that continues to grow. To review the field is not the present objective, because

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Nomenclature

a,b,c	thicknesses, m
C	costs, Eq. (43)
D	transversal dimension, m
f_1	factor, Eq. (1)
H	height, m, Fig. 1
k	fluid thermal conductivity, W m^{-1} K^{-1}
L	length, m, Fig. 1
L_{flow}	flow length, m
L_{long}	long side of cross section, m
L_{scale}	length scale, m
K	local pressure loss factor, Eq. (48)
$K_{P,HT}$	relative cost factors, Eqs. (45) and (46)
$\dot{m}$	mass flow rate, kg s^{-1}
n	number of channels
$\tilde{P}$	pumping power, dimensionless, Eq. (26)
Pr	Prandtl number
q	heat transfer rate, W
r	factor, Eq. (28)
Re_D	Reynolds number
t	wall thickness, m
T	temperature, K
U	mean velocity, m s^{-1}
V	volume, m^3
W	spacing, m, Fig. 1
$\dot{W}$	pumping power, W
X	length, m, Fig. 1

Greek symbols

ΔT	temperature difference, K
η	aspect ratio, X/L
μ	viscosity, kg s^{-1} m^{-1}
ν	kinematic viscosity, m^2 s^{-1}
ξ	aspect ratio, H/L
σ	solid volume fraction

Superscript

$\tilde{()}$	dimensionless

Subscript

HT	heat transfer
M	material
P	pumping
TOT	total

reviews appear regularly in the literature. There are two objectives in this article:

The first objective is to draw attention to the rapid growth of the evolutionary design field toward industrial applications. Advances are published regularly for heat exchangers [4–13], fuel cells [14], fluid networks [15–20], steam generators [21,22], fluid channels [23–27], energy storage [28], transportation [29–32], power generation [33–37], and management [38].

The second objective is to project on this background the concept of constructal dendritic heat exchanger [4,5], and to propose a conceptual advance in design, which applies at all scales, in every channel. For example, the concept proposed here is relevant to the design of evaporator heat exchangers for the geothermal industry, in which currently shell-and-tube types are used. Novel heat exchangers are needed to increase the brine effectiveness and decrease the costs of geothermal power generation.

2. Entrance-length channels

Recent advances in constructal design indicate that a flow channel packs maximum heat transfer density when the wall boundary layers meet at the channel exit (for a review, see Ref. 39). This holds for forced convection and natural convection in both laminar and turbulent regimes. This design feature means that the length of the flow channel must match the entrance length of the flow.

Another way to recognize this design rule is to recognize that the channel dimensions must be fitted to the length scales of the flow that fills the channel. The length scale of the flow field is the thickness of the boundary layer that develops along each surface, downstream from the entrance. For clarity, assume that the velocity boundary layer is nearly as thick as the thermal boundary layer, which means that $Pr \sim 1$. The longitudinal flow length where the boundary layers of opposing surfaces merge is the entrance length. If the actual length of the channel is shorter than the entrance length, then the core of the flow does not participate in convecting heat from the walls, and the channel volume is not 'packed' as densely as possible with heat flow. In the opposite extreme, when the channel length is much greater than the entrance length, most of the volume is filled with fluid that has already been heated or cooled by the walls, exhibiting much lower heat transfer rates from the walls. The maximum heat transfer rate per unit volume belongs to the design located at the *intersection of asymptotes* [39], which is the design with flows that sweep solid surfaces as long as the entrance length of the flow itself.

The flow structure with channels as long as the entrance length differs sensibly from other structures. We showed this fact in the design of a T-shaped construct of two round tubes [40], one stem (D_1, L_1) and two branches (D_2, L_2), on a fixed area ($2L_2L_1$) and subject to fixed total tube volume. When the flow is laminar and fully developed in every tube, the architecture with the smallest overall pressure drop (or smallest pumping power) is characterized by the aspect ratios $D_1/D_2 = 2^{1/3}$ and $L_1/L_2 = 2^{1/3}$. If we add the requirement to maximize the volumetric heat transfer rate, which means that L_1 and L_2 are the entrance lengths of the stem and the branches, the optimal ratio L_1/L_2 becomes 2. The design for minimum pumping power and maximum heat transfer density, simultaneously, is characterized by $D_1/D_2 = 2^{1/3}$ and $L_1/L_2 = 2$, not by $D_1/D_2 = 2^{1/3}$ and $L_1/L_2 = 2^{1/3}$.

3. Dendritic cross-flow plate heat exchanger

In this paper, we show how the entire architecture of a three-dimensional cross-flow plate heat exchanger is determined as a consequence that in every one of its channels, large or small, the channel length scale matches the flow entrance length scale. Specifically, the cross-flow plate heat exchanger shown in Fig. 1 is considered in this paper. Cold fluid ($\dot{m}$) enters from the left through a channel of length L, width H, and spacing a. The stream is distributed to its left side to n parallel-plates channels, each of length X, spacing W, width H, and mass flow rate $\dot{m}/n$.

Fig. 1 is a dendritic flow architecture because it consists of two combs perpendicular to each other, with their bristles in cross flow. The hot stream is one comb, and the cold stream is the other comb. Each comb is a tree with branches on one side only. Previously, we evolved the constructal design of two flow combs matched tip to tip, in the same plane, with application to vasculatures for self-healing and self-cooling (Ref. [40], ch. 8). In Fig. 1, the planes of the two sets of bristles are perpendicular. The first work of this

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