International Journal of Heat and Mass Transfer 114 (2017) 8–19

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Natural convection under sub-critical conditions in the presence of heating non-uniformities

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ARTICLE INFO

Article history:
Received 23 December 2016
Received in revised form 30 March 2017
Accepted 5 June 2017

Keywords:
Natural convection
Subcritical conditions
Heat transfer intensification
Secondary convection

ABSTRACT

When the heating intensity is below the critical threshold required for the onset of Rayleigh-Bénard (RB) convection, heat transport across a horizontal fluid layer uniformly heated from below is driven by conduction, placing a limit on its magnitude. A methodology to increase the heat flow through the use of spatial heating non-uniformities is proposed. The non-uniformities create convection whose pattern is dictated by the pattern of the heating, and this convection supplements the conductive heat transport. It has been shown that the effectiveness of this convection, the primary convection, is a strong function of the heating wave number and it may increase the heat flux by up to ten times when compared with the conductive state if the most effective heating wave number is used. The primary convection is subject to transitions to various secondary states and the critical conditions for these transitions have been determined using linear stability theory. Three modes of secondary convection have been identified giving rise either to rolls parallel to the primary rolls, to rolls orthogonal to the primary rolls, or to oblique rolls. Conditions leading to their onset define the limits on the heat transfer predictions based on the analysis of the primary convection. In general, transition to secondary convection occurs at smaller Rayleigh numbers in the presence of heating non-uniformities than those required for the onset of RB convection, however, under certain conditions these non-uniformities increase the critical Rayleigh number.

1. Introduction

Heat transport in a stationary fluid layer subject to spatially-homogeneous heating from below relies on conduction for small enough heating intensities and is supplemented by convection, the so-called Rayleigh-Bénard (RB) convection, when the heating intensity exceeds a certain threshold. The critical conditions required to achieve this transition are well known [1,2]. There are numerous applications when only a low heating intensity is available and the system cannot breach the RB threshold. This analysis is focused on the search for alternative heat transfer methods suitable for increase of the heat transfer above the conductive limit when the heating conditions are characterized by subcritical values of the Rayleigh number and, accordingly, we refer to this class of problem as heat transfer under subcritical conditions.

It is known that heat transport can be significantly increased by fluid mixing and this mixing can be created either using mechanical mixers or can occur naturally. The latter is preferable as it avoids mechanical devices, is likely the most energy efficient and certainly most reliable as no breakdowns of mechanical systems are possible. Natural mixing under subcritical condition can be achieved using heating non-uniformities as they always produce convection. The goal of this analysis is therefore to identify the most promising spatial distributions of such non-uniformities from the point of view of maximization of the heat flux. The required non-uniformities can be easily created in applications using properly distributed discrete heat sources, e.g. heating wires. Such systems involve a mixture of homogeneous and non-uniform heating and characterizing their performance is of interest.

The character of the system response to the superposition of homogeneous and inhomogeneous heating cannot be predicted a priori due to nonlinearity of the relevant processes. The former case represents a classical problem studied for over one hundred years, starting with Bénard [1] and Rayleigh [2]; see Bodenschatz et al. [3], Ahlers et al. [4], Lohse & Xia [5] and Chilla & Schumacher [6] for recent reviews. RB convection has the form of rolls (striped pattern) and is characterized by a linear neutral stability curve with a well-defined minimum which identifies the critical Rayleigh number and the critical wave vector with the onset conditions being independent of the Prandtl number *Pr*. Explicit estimates for the maximum heat transfer have recently been provided [7]. The use of more intense heating leads to secondary bifurcations and the onset of either the skewed-varicose instability, the

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.06.016
0017-9310/

Eckhaus instability or the zig-zag instability [8,9] described using the so-called Busse balloon. Linear stability theory does not offer any guidelines regarding the spontaneous pattern selection which, in most experiments, is determined by secondary effects. There is a plethora of possible secondary effects with most of the existing analyses focused either on the small boundary temperature inhomogeneities or on the small geometry imperfections with the common goal of explaining experimental findings. Various forms of symmetry breakup have been demonstrated in the former [10,11] as well as the latter [12] cases. The objectives of these analyses consisted of determining variations of the critical conditions and topology of the resulting motions in response to small imperfections rather than on the use of large imperfections to alter the heat transfer process in a qualitative sense. Turbulent convection with imperfections in the form of surface roughness has been subject to extensive analyses from the point of view of increase of heat transfer [13–15] as well as a route for a more rapid approach to the ultimate regime of convection [16]. Studies of various forms of geometric confinement resulted in the identification of interesting methods for intensification of heat transfer [17–22] and have led to geometry optimization [23]. Analysis of horizontal convection [24] represents an alternative route as introduction of surface nonuniformities leads to the generation of horizontal temperature gradients even in the case of nominally isothermal horizontal walls [25].

Detailed studies of convection created by an inhomogeneous heating are very recent. Such heating creates horizontal temperature variations which lead to different vertical pressure distributions at different horizontal locations giving rise to horizontal pressure gradients; these configurations are statically unstable resulting in a fluid movement regardless of the intensity of the heating and leading to motions frequently referred to as horizontal convection [26–29]. The use of spatial patterns of heating leads to the concept of structured convection studied in the case of periodic heating by Hossain & Floryan [30–32]. The topology consists of sets of counter-rotating rolls whose size is dictated by the heating wave number. These rolls completely fill the fluid layer for small and O(1) heating wave numbers but concentrate next to the heated plate for large heating wave numbers. In the latter case, convection is confined to a thin boundary layer adjacent to the heated plate while the temperature field separates into two zones. The temperature does not change in the horizontal direction in the upper zone and the heat is driven across this zone by conduction only. The lower zone, the boundary layer, is characterized by a complex temperature distribution with the heat driven across this zone by convection. The character of transition to secondary states depends strongly on the heating wave number and the system response results from a competition between the RB mechanism and the spatial parametric resonance. In the case of large heating wave numbers, the RB mechanism dominates [30] resulting in the formation of rolls with the wave vector parallel to the wave vector of the primary rolls. When the heating wave numbers are O(1), the spatial parametric resonance dominates, resulting in the formation of oblique rolls with the wave vector having a component orthogonal to the wave vector of the primary convection; such oblique structures are observed in pattern forming systems where they lead to three-dimensionalization of the otherwise two-dimensional flow problem. Wave number locking between the primary and secondary convection occurs under certain conditions. When the heating wave numbers are small, the RB mechanism dominates again leading to the formation of rolls with the wave vector parallel to the wave vector of the primary rolls. Such rolls appear only around the hot spots with no convection taking place in the remaining part of the slot [33]. In all these cases, the onset conditions strongly depend on *Pr*. Heating of the upper plate results in a very similar system response [32]. The heat transfer resulting from periodic heating has been discussed in [31]. A qualitatively similar problem with spatial variability resulting from the use of mixed insulating and conducting boundary conditions has been studied by Ripesi et al. [34] and Marcq & Weiss [35].

Convection resulting from a mixture of uniform and non-uniform heating should exhibit a combination of features discussed above, with the RB features prevailing when the periodic heating is weak and the opposite happening when the uniform heating is weak. While the magnitude of the heat flux resulting from transition between these two limits is the primary interest in this analysis, the changes in the character of convection represent by themselves an interesting fundamental problem with the formation of the secondary states being especially intriguing. Identifying conditions resulting in the formation of such states is necessary as they define the limits of applicability of the heat transfer predictions provided by the primary convection.

The analysis presented below is focused on fluids with $Pr = 0.71$. Section 2 discusses the primary convection, i.e. convection directly driven by heating non-uniformities. Section 2.1 describes a simple model problem which captures the main features of the proposed heat transfer augmentation strategy and provides the means for describing the mechanics of the processes involved. The proposed model consists of a horizontal layer subject to a combination of uniform and non-uniform heating from below resulting in a sinusoidal temperature distribution along the lower plate with its mean higher than the temperature of the isothermal upper plate. The intensity of the uniform part of the heating is characterized by the classical Rayleigh number which is based on the difference between the mean temperatures of the plates. The non-uniform part of the heating is characterized by the periodic Rayleigh number which is based on the amplitude of the sinusoidal temperature variations and its spatial distribution is characterized by the heating wave number. Section 2.2 describes the numerical method used to study the primary convection. Section 2.3 describes the properties of this convection and the resulting heat transfer characteristics. As the primary convection may undergo transition to secondary states, Section 3 is focused on the determination of the onset conditions required for the formation of such states using linear stability theory. The problem formulation for the stability problem is presented in Section 3.1. A description of the secondary states is given in Section 3.2. Section 4 provides a short summary of the main conclusions.

2. Primary convection

Introducing horizontal temperature gradients results in convective fluid movement. As there is an uncountable number of possible temperature distributions, we focus our attention on spatially-periodic heating as its analysis provides a convenient reference point as well as basic information about the patterned convection.

2.1. Problem formulation

Consider fluid contained in a slot between two parallel plates extending to $\pm\infty$ in the x- and z-directions and placed at a distance 2 h apart from each other with the gravitational acceleration g acting in the negative y-direction, as shown in Fig. 1. The upper plate is kept isothermal with the constant temperature T_R while the lower plate is subject to a heating resulting in its temperature T_L being of the form

$$T_L(x) = T_U + \frac{1}{2} T_p \cos(\alpha x) \tag{2.1}$$

where $T_U > T_R$, T_p is the amplitude of the periodic component whose wavelength is $\lambda = 2\pi/\alpha$ where α denotes its wave number. The fluid