International Journal of Heat and Mass Transfer 114 (2017) 135–144

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

A rigorous proof for the uniformity principle of temperature difference field in heat exchanger with deductive method

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ARTICLE INFO

Article history:
Received 18 January 2017
Received in revised form 3 June 2017
Accepted 4 June 2017

Keywords:
Heat exchanger
Uniformity principle of temperature difference field
Universal uniformity factor
Flow arrangement
Thermal conductance

ABSTRACT

In this paper, the uniformity principle of temperature difference field (TDF) in heat exchanger has been proved with deductive reasoning. A new parameter which is used to measure the uniformity of TDF in heat exchangers is defined as the universal uniformity factor instead of the two-dimensional factor. An infinitesimal change of flow arrangement is studied, and the corresponding variations of the uniformity factor and the total thermal conductance are calculated. By analyzing the relation between the variations, it is proved that the more uniform the distribution of TDF, the smaller the total thermal conductance of heat exchanger for given heat transfer rate and inlet parameters of fluid. The deduction proves that the uniformity principle of TDF is strictly true in heat exchangers. An optimization problem is analyzed to show the better applicability of the universal uniformity factor compared with the spacial uniformity factor, and it illustrates that the uniformity principle of TDF applies to the heat exchangers with variable thermal conductivity.

1. Introduction

Heat exchangers are widely used in space heating, refrigeration, power stations, chemical plants, etc. In order to improve the performance of heat exchanger, especially increase the effectiveness of heat exchanger, a great deal of research has been studied [1–8]. The study of heat exchangers has a long history, and the optimization design for heat exchanger is still an important research field because of a series of new requirements.

The heat transfer enhancement methods are generally classified into three categories: Active method, Passive method and Compound method. The active method [9] involves external power input to enhance the heat transfer capacity, while the passive method [10] uses surface or flow arrangement modifications to develop the performance instead of external power, and the combination of these two methods is the compound method [11]. Many researchers have studied the essential factors which effect the heat transfer performance of heat exchangers [12–16]. Of the numerous studies of developing the performance of heat exchanger, Guo *et al.* [17] proposed the uniformity principle of temperature difference field (TDF) in heat exchanger to reveal the essential determinant of the heat exchanger performance, and it has been sufficiently verified by some asymptotic and experimental methods [18], and the principle has been applied to improve the performance of heat exchangers. Cabezas-Gmez *et al.* [19] proposed a new configuration of cross-flow heat exchanger relying on the uniformity principle of TDF, and characterized the new flow arrangement, and the result shown that the new structure is more efficient. Some researchers [20,21] analyzed and optimized the performance of the shell and tube heat exchanger, and used the uniformity principle of TDF to explain the inherent reason of the performance changes. As the extension of the uniformity principle of TDF in heat exchanger, Ren *et al.* [22] indicated that the uniformity principle also applied for the mass transfer potential in a mass exchanger, and developed the performance of internally cooled or heated liquid desiccant-air contact units. On the other hand, Zhang *et al.* [23] proposed a new method to synthesize heat exchanger networks based on the uniformity principle, and applied the uniformity factor in the evaluation of heat exchanger network. Unfortunately, the uniformity principle of TDF is found out from experimental exploration and numerous numerical instances, and there has not been a rigorous proof for the principle because of the complexity in mathematics.

In this paper, the uniformity factor of TDF is redefined with a general form, and the uniformity principle of TDF is stated with an equivalent proposition. An infinitesimal change of flow arrangement is built, and it is proved that any flow arrangement change which leads to a more uniform TDF will lead to a better heat transfer performance. With a rigorous deductive reasoning, this paper

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2017.06.012

Nomenclature

α, β	proportionality coefficient
$\dot{H}$	enthalpy flow rate
$\dot{m}_c$	mass flow rate of cold fluid
$\dot{m}_h$	mass flow rate of hot fluid
$\dot{Q}$	cumulative value of heat transfer rate
$\dot{Q}_0$	total heat transfer rate of heat exchanger
Φ	two-dimensional uniformity factor of TDF
σ	standard deviation
φ	universal uniformity factor of TDF
$c_{p,c}$	constant pressure specific heat capacity of cold fluid
$c_{p,h}$	constant pressure specific heat capacity of hot fluid
E	expectation
A	heat transfer area
K	thermal conductivity
L	dimension of heat exchanger in length direction
t	temperature of cold fluid
T	temperature of hot fluid
W	dimension of heat exchanger in width direction

illuminates that the uniformity principle of TDF is strictly true in heat exchangers. Finally, an instance shows the applicability of the uniformity principle of TDF on the heat exchanger with variable thermal conductivity.

2. The uniformity principle of TDF in heat exchanger

The uniformity principle of TDF in heat exchanger is proposed [17] and confirmed [18] to improve the heat transfer performance of heat exchangers, and it reveals the inherent relationship between the effectiveness of heat exchanger and the uniformity of TDF. For a heat exchanger, the principle is expressed as: the more uniform the TDF, the higher the effectiveness of the heat exchanger for a given number of heat transfer units, NTU, and a fixed heat capacity rate ratio, C_r, which equals the ratio of the larger heat capacity rate of the fluid to the smaller heat capacity rate.

2.1. The two-dimensional uniformity factor of TDF

In the above statement of uniformity principle of TDF, the effectiveness of heat exchanger is a useful parameter to evaluate the performance of heat exchanger, but there is no parameter to measure the uniformity of TDF quantitatively. In order to describe the uniformity of TDF, Guo *et al.* [18] proposed a concept of uniformity factor of TDF, Φ, and defined it with an explicit expression for a two-dimensional problem shown in Fig. 1 [18]. The two-dimensional uniformity factor of TDF is expressed as:

$$\Phi = \frac{\int_0^W \int_0^L [T(x,y) - t(x,y)]\mathrm{d}x\mathrm{d}y}{\sqrt{WL \int_0^W \int_0^L [T(x,y) - t(x,y)]^2 \mathrm{d}x\mathrm{d}y}} \tag{1}$$

where $T(x,y)$ and $t(x,y)$ represent the temperature distributions of hot fluid and cold fluid, respectively; L and W are the dimensions of the heat exchanger. Furthermore, the factors for some typical heat exchanger designs (counter-flow, parallel-flow and cross-flow) have been calculated [17].

2.2. The universal uniformity factor of TDF

In fact, the uniformity factor of TDF defined in Eq. (1) has a great limitation to describe the uniformity of TDF. It can only describe the spacial uniformity of TDF and results in harsh applying conditions of the principle. For heat exchangers with variable thermal conductivity, the spacial distribution of TDF cannot describe the distribution of heat exchange capability completely. The flow arrangement with a uniform spacial distribution of TDF will waste the heat exchange capability of the parts with high thermal conductivity, and these result in the performance degradation of heat exchanger. Therefore, the uniformity principle of TDF with spacial uniformity factor is no longer true if the thermal conductivity is not a constant in the heat exchanger.

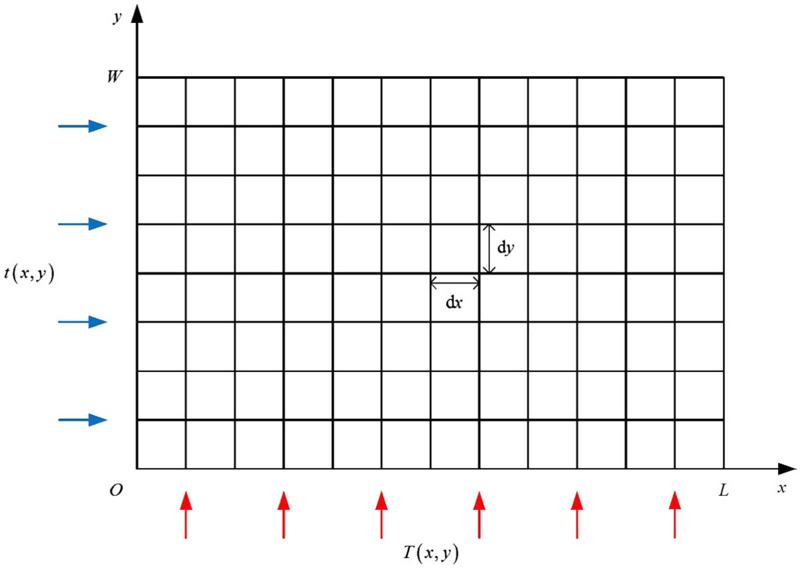

Fig. 1. The two dimensional grid of a cross-flow heat exchanger.