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A triple reciprocity method in Laplace transform boundary element method for three-dimensional transient heat conduction problems

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ABSTRACT

In this paper, a boundary element method employing the Laplace transform is developed to solve the three-dimensional transient heat conduction problems. The fundamental solution for the modified Helmholtz equation is adopted in order to derive the basic integral equations. Due to the effects of initial temperature and heat generation, the domain integrals appearing in the integral equation will degrade the advantages of boundary element method. Therefore, a new triple reciprocity formulation in Laplace domain is proposed to convert the domain integrals into boundary integrals. The higher order fundamental solutions required in the triple reciprocity method can be obtained from the original fundamental solution by multiplying a constant term, which leads to a much simpler formulation in the Laplace domain. Then, the inverse transformation can be used to obtain the solutions in the time domain. Several numerical examples are presented to demonstrate the efficiency and accuracy of the proposed method. In summary, the triple reciprocity formulation is a useful approach to capture the transient heat conduction responses.

1. Introduction

Transient heat conduction problems can be efficiently solved by the boundary integral equation (BIE) method [1]. The various solution procedures reported in the literature can essentially be classified into two broad categories: the time domain approach [1–4] and the transform space approach [5–7]. Without the presence of heat source and non-uniform initial temperature distributions, transient heat conduction problems can be easily solved using the conventional boundary element method (BEM) and internal cells are not needed. However, the domain integral becomes necessary when initial temperature distribution is not uniform and the heat generation function is arbitrary. Both aforementioned approaches involve the time-consuming domain integral calculations. Therefore, the advantage of dimensionality reduction is lost in these BEM based methods. In order to avoid the domain integration, several transformation methods of the domain integral have been proposed, including combining time-domain methods and the transform space methods [8–12]. All those methods exhibit different characteristics.

The implementation of the time-domain approach is straightforward. The time-dependent fundamental solution (FS) is generally used as the weight function to derive the BIE [1–4]. As discussed in the literatures about the transformation method of the domain integral, the Green's function from the Laplace equation is always employed [13–18]. The dual reciprocity method (DRM), developed by Nardini and Brebbia, is typically used transformation method, which is based on a set of interpolation functions and their particular solutions [8,9]. Singh applied the DRM to transform the domain integral, which is associated with the time derivative of temperatures, to obtain transient diffusion analysis [13]. Tanaka et al. introduced the DRM to solve the transient heat conduction problems with both homogeneous and inhomogeneous materials, in which the time derivative is approximated by the time-stepping method [14]. Nowak and Neves developed the multiple reciprocity method (MRM) to convert the domain integral to an infinite series of boundary integrals [15]. For the transient heat conduction problem, the higher-order FS of the Laplace equation is used and either linear or quadratic approximations are applied to the time derivative [10]. The radial integration method (RIM) is a new transformation approach which was developed by Gao [12]. This new approach not only can transform any complicated domain integrals to its boundaries without using particular solutions, but also can remove various singularities appearing in the domain integrals. Yang applied the RIM to solve transient heat

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conduction problems with varying heat conductivities [16]. Based on the RIM, Feng presented a single integral equation method to solve the general multi-medium problems [17]. Furthermore, an analytical expression of the radial integral was derived for efficient computation [18]. The time-dependent FS is used as discussed in these works [19–22]. Chen and Wang applied the singular boundary method (SBM) to transient diffusion equations. This method involves boundary only and is free of integration and mesh [19]. Wang and Chen presented simple empirical formulas for the evaluation of the origin intensity factors (OIFs) in the SBM for transient diffusion problems [20]. In addition, an analytical evaluation of the OIFs was conducted in three dimensional cases by Wang et al. [21]. Yoshihiro Ochiai proposed the triple reciprocity method (TRM), in which the pseudo-initial temperature and/or heat sources density are approximated by using the triple-reciprocity formulation [22]. The time-dependent FS and its higher order forms are employed and their expressions are complex. Finally, the domain integrals are converted to the boundary integrals. Only the boundary elements and interior free-scattering points are needed in the discretization model.

In the transform space approach, the time dependent issue can be temporarily remedied using the Laplace transform. The parabolic heat conduction equation is transformed into a more tractable elliptic equation (also called the modified Helmholtz equation). Then several BIEs, which contain different transformation parameters, are derived and solved in the Laplace space. Finally, the inverse transformation is performed to evaluate the physical variables in the real space [5]. Based on the Laplace transform approach, Sutradhar et al. analyzed the transient heat conduction problems with both homogeneous and non-homogeneous materials using the Galerkin BEM [6,7], in which the FS of the modified Helmholtz equation is used. Zhu applied the DRM to transform the domain integral associated with both the temperature variation and the non-uniform initial temperature distribution [23]. Amado et al. studied the applicability of the Laplace transform based dual reciprocity BEM for laser heat treatment model and included the nonlinear formulations due to temperature dependent material properties [24]. Yu et al. derived a new method formed by the differential transformation method and the radial integration BEM to solve transient heat conduction problems with functionally graded materials [25]. The FS of the Laplace equation is used in order to derive the integral equation as discussed in above works [23–25]. Guo developed a new multiple reciprocity formulation to solve the three-dimensional transient heat conduction problems [26]. First, the FS of the modified Helmholtz equation is used to derive the BIE. Then, the MRM, in which the higher order FSs of the modified Helmholtz equation and the analytic high order derivatives of the domain functions are required, is employed to transform the domain integrals to boundary integrals. Finally, two examples of 3-D (three-dimensional) transient heat conduction problems, which contained polynomial type initial temperature distribution and sine type heat generation respectively, are solved by the proposed method and the numerical results with good accuracy are obtained. Therefore, the analytic functions of the initial temperature distribution and heat generation are necessary in MRM. However, in many engineering problems, the distribution of initial temperature is obtained by sampling at monitoring point and cannot be expressed analytically. Thus it is hard to get the high order derivatives of the initial temperature.

In this paper, the Laplace transform triple reciprocity boundary element method (LT-TRBEM) is employed to solve the transient heat conduction problem containing uniform initial temperature distribution or heat generation, in which the analytical functions are not necessity. First, the Laplace transform methodology was applied to eliminate the time-dependent issue. The FS of the modified Helmholtz equation was used to derive the BIE in Laplace domain. Secondly, the high derivatives of the initial temperature field or the heat generation, which were necessities in the TRM, were computed by solving a system of equations using original values of them at the discretized points. Thirdly, the domain integrals were converted to the equivalent boundary integrals by employing the TRM, which overcame the limitations of the MRM. The higher order FSs used in TRM could be obtained from the original FS by multiplying a constant term, which made the TRM formulation in Laplace domain much simpler than that in time domain. Finally, the Gaver-Wynn-Rho algorithm, which combined with the Wynn's rho algorithm to improve the convergence rate, was applied to perform the Laplace inversion. Then the results in time domain are obtained. In addition, it should be mentioned that to avoid the differences between the geometric model and the analysis model, our method is implemented in the framework of boundary face method (BFM) program. The BFM is implemented directly based on the boundary representation data structure (B-rep) that is used in most CAD packages for geometry modeling [4,27,28]. Each bounding surface of geometry model is represented as parametric form by the geometric map between the parametric space and the physical space. Both boundary integration and variable approximation are performed in the parametric space. The integrand quantities are calculated directly from the faces rather than from elements, and thus no geometric error will be introduced.

The remainder of this paper is organized as follows. The BIE in Laplace domain is derived briefly in Section 2.1. The interpolation of terms $b(\mathbf{x})$ which involves the domain integrals is presented in Section 2.2. In Section 2.3, the characteristic of the modified Helmholtz FS is presented. The TRM formulation in Laplace domain is presented in Section 2.4. Section 2.5 briefly describes the Laplace inversion and solution procedures. Some numerical examples are presented in Section 3. Finally, concluding remarks and directions for future research are discussed in Section 4.

2. Problem definition

2.1. The BIE in Laplace domain for transient heat conduction

Assuming isotropic material, the 3-D transient heat conduction problem with heat generation can be represented as

$$\begin{aligned} &\nabla^2 u(\mathbf{x},t) = \tfrac{1}{a}\tfrac{\partial u(\mathbf{x},t)}{\partial t} - \tfrac{1}{k} w(\mathbf{x},t). && \mathbf{x}\in\Omega \\ &u(\mathbf{x},t) = \bar{u}(\mathbf{x},t). && \mathbf{x}\in\Gamma_1 \\ &q(\mathbf{x},t) = \tfrac{\partial u(\mathbf{x},t)}{\partial n(\mathbf{x})} = \bar{q}(\mathbf{x},t). && \mathbf{x}\in\Gamma_2 \\ &u_0(\mathbf{x}) = u(\mathbf{x},t_0). && \mathbf{x}\in\Omega \end{aligned} \tag{1}$$

where $u(\mathbf{x}, t)$ is the temperature at location $\mathbf{x}$ and time t. $w(\mathbf{x}, t)$ is the heat source density. The coefficient a and k are the thermal diffusivity and the heat conductivity respectively. Ω stands for the considered domain enclosed by $\Gamma_1 \cup \Gamma_2$. $\bar{u}$ and $\bar{q}$ stand for the prescribed temperature and normal flux on the boundary respectively. $u_0(\mathbf{x})$ is the initial conditions at time $t = t_0$.

The Laplace transform of the function $u(\mathbf{x}, t)$ in denoted by

$$\tilde{u}(\mathbf{x},s) = L(u(\mathbf{x},t)) = \int_0^\infty u(\mathbf{x},t)e^{-st}\,dt \tag{2}$$

Assuming that the transformation parameter s is real and positive, the governing equation in the Laplace domain is given by

$$\begin{aligned} &\nabla^2 \tilde{u}(\mathbf{x},s) - \tfrac{s}{a}\tilde{u}(\mathbf{x},s) + \tfrac{1}{a}u_0(\mathbf{x}) + \tfrac{1}{k}\tilde{w}(\mathbf{x},s) = 0 && \\ &\tilde{u}(\mathbf{x},s) = \bar{u}(\mathbf{x},s). && \mathbf{x}\in\Gamma_1 \\ &\tilde{q}(\mathbf{x},s) = \bar{q}(\mathbf{x},s). && \mathbf{x}\in\Gamma_2 \end{aligned} \tag{3}$$

Above equation is also called the modified Helmholtz equation. $\bar{u}(\mathbf{x},s)$ and $\bar{q}(\mathbf{x},s)$ define the function values on the boundaries.