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A simple difference method for lattice Boltzmann algorithm to simulate conjugate heat transfer

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ABSTRACT

In the present paper, a simple difference method for lattice Boltzmann algorithm is proposed to simulate conjugate heat transfer problems. In the conventional lattice Boltzmann method (LBM), the informations including temperature and heat flux exchange directly between two different media through distribution function during the streaming process, however, the continuity of heat flux at the interface between two different media cannot be guaranteed in this process. Different with the conventional LBM, we consider that the nodes near the interface get the distribution functions from the interface during the streaming process across the interface. The distribution functions at the interface can be obtained by coupling the interface conditions of temperature and heat flux with non-equilibrium extrapolation. Four test cases are used to validate the present method, including both steady and transient conjugate heat transfer with flat or curved interfaces. The results show that the present method is very easy to implement, and feasible for both steady and transient heat transfer problems. In addition, for simplicity, by approximating the real interface with a staircase shaped line, the present method can deal with curved interface easily and the results show the approximation will not contribute obvious error to the final results.

1. Introduction

Conjugate heat transfer is a very common physical phenomenon which relates to numerous applications in engineering, such as the cooling of electronic devices [1], electronic equipment [2], the heat transport in micro fuel cells [3] and microchannels [4,5], etc. Depending on the numerical methods for solving the governing N-S and energy equations, various approaches have been proposed to simulate conjugate heat transfer between two domains of solids, solid and fluid, or fluids [6–8] with different physical properties. The conventional numerical methods, such as Finite Volume Method (FVM), Finite Difference Method (FDM), and Finite Element Method (FEM) have been successfully applied to simulate conjugate heat transfer problems. For instance, Fiebig et al. [9] made a series of numerical simulations on conjugate heat transfer in a finned-tube element with FVM. Ha and Jung [10] simulated three-dimensional natural convection and conduction problems in a differentially heated cubic enclosure with a heat-generating cubic conducting body with FVM. By taking FDM, Korichi and Oufer [11] made a numerical investigation on solid-fluid conjugate heat transfer in a rectangular channel with discrete obstacles on upper and lower walls. More systematic introduction about conjugate heat transfer problems can be found in the review written by Dorfman and Renner [12] and the references therein.

Based on the evolution of particle distribution functions of discrete velocities, the lattice Boltzmann method (LBM) has been developed rapidly in recent years. As proved by some previous researchers, LBM is a very powerful and simple method in simulating the multiphase flows in fluid-fluid systems [13–16] and solid-fluid systems [17–20]. Up to date, some researchers have tried to spread the use of LBM from multiphase flows simulation to conjugate heat transfer simulations. Since the conventional LBM can only retrieve the standard energy equation when the two different media have the same heat capacities, some modifications are necessary for more general cases when the heat capacities of the two different media are unequal. For instance, an improved SIMPLE-like algorithm has been adopted by some researchers to simulate conjugate heat transfer problems [21–23]. This method assumes that the two different media have the same heat capacities ρc_p (ρ and c_p are, respectively, the density and the specific heat capacity at constant pressure), therefore, is only capable for conjugate heat transfer at steady state. Also, a kind of difference method has been proposed [24] to deal with the conjugate heat transfer problems, in which the continuous heat flux at the interface between two different media can be well guaranteed. However, this method is

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Nomenclature

c_L	lattice speed (m/s)
c_p	specific heat at constant pressure $(\mathrm{kJ/(kg\cdot K)})$
c_s	lattice sound speed (m/s)
$\vec{e}_i$	discrete velocity in direction i (m/s)
f_i	density distribution function $(\mathrm{kg/m^3})$
$f_i^{(eq)}$	equilibrium density distribution function $(\mathrm{kg/m^3})$
F_i	discrete body force term $(\mathrm{kg/m^2\ s^2})$
$\vec{F}$	body force term $(\mathrm{kg\cdot m/s^2})$
$\vec{g}$	gravitational acceleration $(\mathrm{m/s^2})$
g_i	energy distribution function (K)
$\hat{g}_i$	energy distribution function after collision step (K)
$g_i^{(eq)}$	equilibrium energy distribution function (K)
g_i^{neq}	non-equilibrium energy distribution function (K)
H	height (m)
k	heat conductivity $(\mathrm{W/(m\cdot K)})$
N	grid number in y axis
$\overline{Nu}$	average Nusselt number
Pr	Prandtl number $Pr = \nu/\alpha_f$
Q	heat source term $(\mathrm{W/m^3})$
Q_i	discrete heat source term in direction i $(\mathrm{W/m^3})$
$\vec{q}$	heat flux $(\mathrm{W/m^2})$
r	distance to central point (m)
R	radius (m)

Ra	Rayleigh number $Ra = g\beta(T_h - T_c)H^3/\upsilon\alpha_f$
T	temperature (K)
T_h	hot temperature (K)
T^a	temperature of analytical solution (K)
T_c	cold temperature (K)
$\vec{u}$	velocity (m/s)
w_i	weight coefficients in direction i for D2Q5 model
x	abscissa (m)
y	ordinate (m)

Greek symbols

α_{ij}	thermal diffusivity $(\mathrm{m^2/s})$
α_f	thermal diffusivity of fluid $(\mathrm{m^2/s})$
β	volume expansivity (1/K)
ρ	density $(\mathrm{kg/m^3})$
ν	kinematic viscosity $(\mathrm{m^2/s})$
θ	dimensionless temperature of $\theta = (T - T_c)/(T_h - T_c)$
ω_i	weight coefficients in direction i for D2Q9 model
Δt	lattice time step (s)
Δx	lattice space (m)
τ_f	relaxation time of density distribution function
τ_g	relaxation time of energy distribution function
φ	the angle with horizontal line

confined to solids with a square shape at steady state. Recently, some improved difference methods were proposed to eliminate these drawbacks. Li et al. [25] proposed a new difference method to ensure temperature and heat flux to be continuous at a curved interface, but the difference formulas are too complex to implement, especially for the cases with complex geometry in three dimension. Another promising difference method was proposed by Mohamad et al. [26], in which the continuity of heat flux at the interface is ensured by coupling temperature difference with a new scaling law of energy distribution function. It is suitable for both steady and unsteady conjugate heat transfer between two different immiscible media, what confines the application of this method is that the interface must locate at the computational nodes. Another relative simple difference method was proposed by Mozafari-Shamsi et al. [27], for a streaming process across the interface in this method, the start node is replaced by a virtual node with the same thermal properties of the end node, the temperature of the virtual node is corrected to keep the continuities of temperature and normal heat flux at the interface. Then, the equilibrium distribution function of the virtual node can be evaluated while the non-equilibrium distribution function is obtained by a non-equilibrium interpolation, and the boundary conditions at the interface can be ensured in this difference scheme.

In addition to the difference method, some other approaches could be applied to deal with a conjugate heat transfer problem. In some researches [28,29], double distribution functions were adopted for two different media respectively. In this method, the interfacial nodes which are the computational nodes representing the geometry of the interface obtain the unknown local distribution functions by combing the known distribution functions, boundary conditions and an assumption of the unknown distribution functions. However, the calculation steps for the unknown distribution functions depend on the type of interfacial nodes, which is not simple to be implemented and harder for curved interfaces. Another thought is to add an additional source term to correct the influence induced by the difference of heat capacities. For instance, Karani and Huber [30] suggested using an additional heat source term to recover the energy equation approximately with a first-order accuracy. This method can also be used to simulate conjugate heat transfer with a curved interface approximately. It is very easy to be implemented, however, introduces the jump conditions at the interface which bring obvious deviation to the final results in the area near the interface. Rihab et al. [31] solved the enthalpy equation instead of the temperature equation to get the temperature field indirectly, which can make the additional source term to be calculated easily. However, their method is only applicable for conjugate heat transfer without convection. Recently, a novel method for solving conjugate heat transfer problems named “virtual heat capacity correction method” was proposed by Lu et al. [32], this method firstly assumes that the solid has the same heat capacity with fluid, like SIMPLE-like algorithm, to ensure the boundary conditions at the interface, then correct the temperature field at the end of each time step to get the right temperature field at transient period. However, the method can keep convergence for conjugate heat conduction but may suffer a stability problem in some solid-fluid systems while the heat capacity of the fluid is larger than that of the solid.

Considering the complex geometry, and the difference of thermal properties between the two sides of an interface, the numerical simulations for conjugate heat transfer are generally not easy to be implemented with LBM. A more accurate, reliable, and simpler lattice Boltzmann algorithm is still desired. Based on the idea of correcting the conventional streaming process to keep the continuity of heat flux, a simple difference method is proposed for lattice Boltzmann algorithm to simulate conjugate heat transfer. In this method, the computational nodes adjacent to the interface do not get the distribution functions from the opposite nodes in another medium but from the interface during the streaming process across the interface. The distribution functions of the interface are obtained by coupling the boundary conditions (both temperature and heat flux are continuous) with non-equilibrium extrapolation.