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The effect of instability flow for two-dimensional natural convection in a square enclosure with different arrays of two inner cylinders

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ABSTRACT

This study numerically investigates the two-dimensional natural convection in a square enclosure with different arrays of two inner cylinders at Rayleigh numbers of $10^3 \le Ra \le 10^6$. A simulation was carried out based on the immersed boundary method to obtain an accurate solution. The results were compared with those of two inner circular cylinders with different vertical locations from a previous study. Detailed analysis results are presented for the distribution of the flow and thermal fields and the time- and surface-averaged Nusselt numbers. The flow and thermal fields eventually reach steady or unsteady states, depending on the variation in the distance between the cylinders.

1. Introduction

Natural convection in an enclosure is relevant to many fields of academia, industrial applications, and tools, such as nuclear and chemical reactors, cooling of electronic equipment, and heat exchangers. Heat transfer by natural convection exhibits a great variety of complex dynamic behaviors, which depend on the temperature difference between the inner bodies and outer bodies, the shapes and sizes of the inner bodies, and their position and arrangement [1–26].

Cesini et al. [1] investigated the influence of the aspect ratio of the enclosure on the natural convection in the enclosure and Kim et al. [7] studied the effect of the position of the inner cylinder on the natural convection in the space between the enclosure and the cylinder. Lacroix et al. [12] investigated the interaction between convection in the fluid-filled cavity and conduction in the vertical walls, indicating that heat transfer is strongly influenced by the coupling effect between solid wall conduction and fluid convection.

A previous study [27] looked at the influence of different vertical locations of two inner cylinders. The cylinders were equally moved in a vertical array, and the Rayleigh number range of 10^3 - $\le Ra \le 10^6$ was investigated. This study concentrated on the effects of the gap between the cylinders on the natural convection characteristics. The flow regime for unsteady state occurred at $Ra = 10^6$ and when the distance between the cylinders was $\delta_v = 0.3L$. The regime was strongly affected by the Rayleigh number and the distance between the cylinders.

This paper is a continuation of that study. The effect of different arrays of cylinders was investigated in the same ranges of the Rayleigh number and Prandtl number. The transition from steady state to an unsteady state was analyzed with horizontal and diagonal arrays of the cylinders. This study focuses on the effects of variation in the arrays and the transition from steady state to unsteady state. In addition, the heat transfer characteristics were investigated at a relatively high Rayleigh number of $Ra = 10^6$. The flow and thermal fields and the time- and surface-averaged Nusselt numbers were analyzed according to the distance between the cylinders, compared with the previous results [27].

2. Computation details

2.1. Numerical methods

The numerical method is exactly the same as that used in the previous study [27]. The immersed boundary method is easier to implement and more efficient than classical approaches such as body-fitted curvilinear grids. Thus, this method was used to handle the surface of the cylinders in the square enclosure. The governing equations are the continuity, momentum, and energy equations in their non-dimensional forms:

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Nomenclature

f_i	momentum forcing
g	acceleration of gravity
L	length of square enclosure
n	normal direction to the wall
Nu	local Nusselt number
$\langle Nu \rangle$	surface-averaged Nusselt number
$\overline{\langle Nu \rangle}$	time- and surface-averaged Nusselt number
P^*	pressure
P	dimensionless pressure $\left(=\frac{P^* L^2}{\rho\alpha^2}\right)$
Pr	Prandtl number $(=\nu/\alpha)$
r	dimensionless radius of the cylinder $(=R/L)$
R	radius of circular cylinder
Ra	Rayleigh number $\left(=\frac{g\beta L^3(T_h-T_c)}{\nu\alpha}\right)$
t^*	time
t	dimensionless time $(=\frac{t^*\alpha}{L^2})$
T	dimensional temperature
T_h	hot temperature
T_c	cold temperature
u_i^*	velocity
u_i	dimensionless velocity $\left(=\frac{u_i^* L}{\alpha}\right)$
x_i^*	Cartesian coordinates
x_i	dimensionless Cartesian coordinates $\left(=\frac{x_i^*}{L}\right)$

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
δ	dimensionless distance between two cylinders
δ_{i2}	Kronecker delta
ρ	density
ν	kinematic viscosity
θ	dimensionless temperature $\left(=\frac{T-T_c}{T_h-T_c}\right)$

Superscripts /Subscripts

$*$	dimensional value
C	cylinder
En	enclosure
h	horizontal array
v	vertical array
d	diagonal array

$$\frac{\partial u_i}{\partial x_i} - q = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + Pr\frac{\partial^2 u_i}{\partial x_j \partial x_j} + RaPr\theta(sin\varphi\delta_{i1} + \cos\varphi\delta_{i2}) + f_i \tag{2}$$

$$\frac{\partial \theta}{\partial t} + u_j\frac{\partial \theta}{\partial x_j} = \frac{\partial^2 \theta}{\partial x_j \partial x_j} + h \tag{3}$$

The dimensionless variables shown above are defined as follows:

$$t = \frac{t^*\alpha}{L^2}, \quad x_i = \frac{x_i^*}{L}, \quad u_i = \frac{u_i^* L}{\alpha}, \quad P = \frac{P^* L^2}{\rho\alpha^2}, \quad \theta = \frac{T - T_c}{T_h - T_c} \tag{4}$$

In these equations, ρ, T, and α represent the density, dimensional temperature, and thermal diffusivity, respectively. The superscript $*$ in Eq. (4) represents the dimensional variables. x_i is the dimensionless Cartesian coordinate, u_i is the corresponding dimensionless velocity component, t is the dimensionless time, P is the dimensionless pressure, and θ is the dimensionless temperature.

The non-dimensionalization results in two dimensionless parameters: $Pr = \nu/\alpha$ and $Ra = g\beta L^3(T_h - T_c)/\nu\alpha$, where ν, g, and β are the kinematic viscosity, gravitational acceleration, and volume expansion coefficient, respectively. The terms q, f_i, and h in Eqs. (1)–(3) are related to the immersed boundary method. The mass source/sink q in Eq. (1) and momentum forcing f_i in Eq. (2) are imposed on the body surface and inside the body to satisfy the no-slip condition and mass conservation in the cell containing the virtual boundary. In Eq. (3), the heat source/sink term h is applied to satisfy the isothermal boundary condition at the virtual boundary. A second-order linear or bilinear interpolation scheme was applied to satisfy the no-slip and isothermal conditions at the immersed boundary. Kim et al. [28], Kim and Choi [29], and Choi [30] describe further details about the immersed boundary method.

The central difference scheme with second-order accuracy based on the finite volume method was used for the spatial discretization of Eqs. (1)–(3). The fractional step method proposed by Choi and Moin [31] was used to simulate the time advancement of the flow field. In the discretization process, the advection terms were treated explicitly using the second-order Adams-Bashforth scheme, and the diffusion terms were treated implicitly using the second-order accurate Crank-Nicolson scheme. Once the velocity and temperature fields are obtained, the local and surface-averaged Nusselt numbers are calculated as follows:

$$Nu = \left.\frac{\partial \theta}{\partial n}\right|_{wall}, \quad < Nu > = \frac{1}{S}\int_0^S Nu\,ds \tag{5}$$

where n represents the direction normal to the wall and S is the length of the surface.

2.2. Computational conditions

Fig. 1 shows the computational domain, its coordinate system, and the boundary conditions. The system consists of a square enclosure with two inner cylinders. The length of each enclosure wall is L, and the radius of the cylinders is $R = 0.1L$. The Prandtl number was set as 0.7, and the Rayleigh number range was $10^3 \leq Ra \leq 10^6$. The cylinders were moved along the horizontal and diagonal directions about the centerline of the enclosure in the range of $0.1L \leq \delta \leq 0.5L$, where δ represents the dimensionless distance between the cylinders in each direction. The cases of δ_h mean that the y positions of the cylinders are fixed and the x positions of the left and right cylinders change along the horizontal centerline. For the cases of δ_d, both the x and y positions of the cylinders change along the diagonal centerline.

Non-dimensional temperatures are imposed on the cold walls ($\theta_c = 0$) of the enclosure and the surfaces ($\theta_h = 1$) of the hot cylinders. No-slip and impermeability conditions are imposed on the surfaces of the cylinders and enclosure walls. All fluid properties are assumed to be constant except for the density in the buoyancy term. The Boussinesq approximation was used to model the variation in the fluid density in the buoyancy term due to the change in the fluid temperature. Gravitational acceleration was applied in the negative y-direction. The grid system was exactly the same as that used in the previous study [27], which was validated by comparing the results of the surface-averaged Nusselt number on the cylinder surfaces in the Rayleigh number range of $10^3 \leq Ra \leq 10^6$. In addition, grid independence of the solution has been tested with additional simulations on much finer grids.