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Numerical analysis for peristalsis of Williamson nanofluid in presence of an endoscope

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ABSTRACT

Main theme of present article is to investigate the peristaltic activity of MHD flow of Williamson nanofluid saturating porous space. Inner tube is fixed while the outer tube is subjected to sinusoidal traveling wave. Unlike the traditional approach, modified Darcy's law characterizes the flow of Williamson fluid. Both tubes with reference to blood arterial flow are considered flexible. No-slip condition for velocity is employed. Convective conditions of heat and mass transfer are invoked. Nanofluid flow using Buongiorno model that considers the effects of both Brownian motion parameter and thermophoresis parameter are also under consideration. The nonlinear differential systems are reduced for large wavelength analysis. Numerical study is performed for the resulting problems. Impacts of various emerging parameters for velocity, temperature, nanoparticle volume fraction and heat transfer rate are examined. A comparative study reveals good agreement with existing limiting study.

1. Introduction

Peristaltic process is now recognized in physiology and modern industry. At present motorized machinery (devices and pumps) is designed using peristaltic phenomenon which transports toxic liquid and sanitary materials. In physiology peristalsis involved spontaneous oscillation of blood vessels, motion of chyme, contraction of walls through which urine passes from kidney to bladder and in many other reproduction systems. Finger and roller pumps are the attractive examples of peristaltic transport in industrial system. Recently the peristaltic motion of many biological fluids (chyme, spermatic fluid and blood) has attracted numerous scientists. These non-Newtonian fluids provide better understanding of peristalsis involved in blood vessels, small intestine and lymphatic vessels. Thus peristalsis has been focussed in the past through different aspects (see Refs. [1–8] and many investigations therein).

Further interaction of magnetic in peristalsis has key role in circulation of blood (as it reduces the blood pressure and bleeding in surgeries). Biomagnetic fluids have great importance in medicine and bioengineering. Some devices are designed through MHD principles. These include MHD compressor operation, blood pump machines, design of heat exchangers, flow meters, power generators, radar systems, etc. Such principles are used for targeted drugs transport, reduction of blood during surgeries, development of magnetic devices for cell separation, development of MHD tracers, cancer therapy, hyperthermia, magneto therapy (which largely involved in MHD non-Newtonian materials) in the field of bioengineering and medical sciences. Furthermore crystal growth, metal casting and liquid metal cooling blankets for fusion reactors are the applications of MHD in industrial sector. Having all such in mind, many researchers made advancements for peristaltic flows of MHD fluids in a channel/tube (see Refs. [9–20]).

Nanofluids are recognized as an auxiliary or a smaller amount of consistent diffusion of unyielding particles (1–100 nm). Nanofluids have higher thermal conductivity and thermal coefficient when compared to base fluids. Inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage, and gravity are the seven modes which produce relative velocity between the tiny particles and base fluid. Out of these seven mechanisms Brownian diffusion and thermophoresis are important slip mechanisms in nanofluid. Such fluids have importance in biochemistry, medicine and engineering fields such as involvement of nanofluid in surgeries, diagnosis and (cancer, vivo, photodynamic) therapies. Also the participation of nanofluid in protein engineering, drug delivery system, neuron electronic interfaces and nonporous materials for size exclusion chromatography cannot be ignored. The awareness of nanofluid has been given by Choi [21] and Buongiorno [22]. Later on many researchers have made

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advancements in flows of nanofluids from various quarters (see Refs. [23–27]).

A porous medium is a material volume consist of solid matrix with an interconnected pores (voids). These voids are completely filled with fluid. The peristalsis through porous medium is relevant for study of transportation of bile via bile duct and urine transport via ureteral tract with stones. Hence flows via porous medium attracted the attention of scientists and engineers in view of therein practical applications in natural and artificial environmental systems. Example of natural porous media are beach sand, filtration of fluids and seepage of water in river beds, petroleum reservoirs, sand, sand stone, limestone and gall bladder with small stones in blood vessels. Theoretically such flows are studied at low Reynolds number with porous media and can be viewed in pores of tubes and capillaries. In pathological conditions, the circulation of cholesterol, blockage and blood clots in blood vessels can be considered similar to porous medium (see Refs. [28–33]). Mostly classical Darcy law is employed to analyze the effect of porous medium even for non-Newtonian fluids. It is not true in reality. The research on peristaltic flow of non-Newtonian fluid via porous space followed by modified Darcy law is limited (see Refs. [34–37]).

Nadeem et al. [38] and Kothandapani and Parkash [39] studied the peristaltic motion of Williamson nanofluid in curved and tapered asymmetric channels respectively. Hayat et al. [40] studied melting heat transfer in flow of Williamson fluid. To our best information the endoscopic relevance in peristaltic motion of Williamson nanofluid saturating porous space is not attended yet. Hence modified Darcy law is employed for the porous space effect relevant to Williamson fluid. Fluid is magnetohydrodynamic by a uniform applied magnetic field. Elastic characteristics of inner and outer tubes are accounted. These tubes also satisfy convective conditions of heat and mass transfer. Mathematical problems are formulated. Numerical solutions are analyzed for embedded parameters involved in the definitions of governing problems. Main observations through discussion are pointed out.

2. Formulation

We intend to investigate the peristaltic transport of an incompressible Williamson nanofluid between two eccentric tubes. Inner tube is considered rigid while a sinusoidal wave of speed c propagates along the wall of outer tube. Nanofluid model consists of Brownian motion and thermophoresis. Fluid is electrically conducting in the presence of uniform applied magnetic field B_0. Induced magnetic field for small magnetic Reynolds number is neglected. An incompressible nanofluid saturates the porous medium. The flexible characteristics of inner and outer tubes are

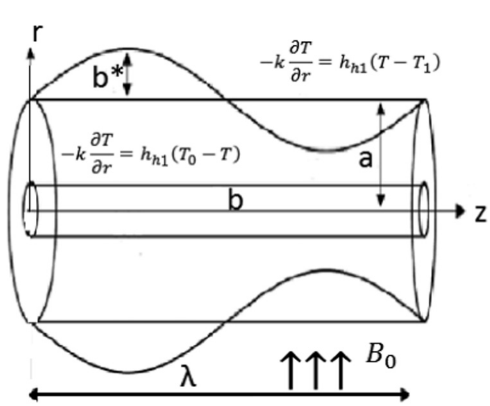

Fig. 1. Flow geometry.

considered. Heat and mass convective conditions at inner and outer tubes are imposed. The equations of inner and outer tubes are (see Fig. 1):

$$\bar{r}_1 = b, \tag{1}$$

$$\bar{r}_2 = a + b^* \sin\frac{2\pi}{\lambda}(\bar{z} - c\bar{t}), \tag{2}$$

in which "a" and "b" are the radii of outer and inner tubes respectively, b^* the wave amplitude, λ the wavelength and t the time.

The expressions describing the present flow are [24]:

$$\frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\bar{u}}{\bar{r}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0, \tag{3}$$

$$\rho_f\left[\frac{d\bar{u}}{dt}\right] = -\frac{\partial \bar{p}}{\partial \bar{r}} + \frac{1}{\bar{r}}\frac{\partial(\bar{r}\bar{S}_{\bar{r}\bar{r}})}{\partial \bar{r}} + \frac{\partial(\bar{S}_{\bar{r}\bar{z}})}{\partial \bar{z}} - \frac{\bar{S}_{\bar{\theta}\bar{\theta}}}{\bar{r}} + \bar{R}_r, \tag{4}$$

$$\rho_f\left[\frac{d\bar{w}}{dt}\right] = -\frac{\partial \bar{p}}{\partial \bar{z}} + \frac{1}{\bar{r}}\frac{\partial(\bar{r}\bar{S}_{\bar{r}\bar{z}})}{\partial \bar{r}} + \frac{\partial(\bar{S}_{\bar{z}\bar{z}})}{\partial \bar{z}} - \sigma B_0^2\bar{w} + \bar{R}_z, \tag{5}$$

$$\left[\frac{d\bar{T}}{dt}\right] = \alpha_f\left[\frac{\partial^2 \bar{T}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right] + \tau D_B\left(\frac{\partial \bar{C}}{\partial \bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}} + \frac{\partial \bar{C}}{\partial \bar{z}}\frac{\partial \bar{T}}{\partial \bar{z}}\right) + \tau\frac{D_{\bar{T}}}{\bar{T}_m}\left[\left(\frac{\partial \bar{T}}{\partial \bar{r}}\right)^2 + \left(\frac{\partial \bar{T}}{\partial \bar{z}}\right)^2\right], \tag{6}$$

$$\left[\frac{d\bar{C}}{dt}\right] = D_{\bar{B}}\left[\frac{\partial^2 \bar{C}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{C}}{\partial \bar{r}} + \frac{\partial^2 \bar{C}}{\partial \bar{z}^2}\right] + \frac{D_{\bar{T}}}{\bar{T}_m}\left[\frac{\partial^2 \bar{T}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{T}}{\partial \bar{r}} + \frac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right]. \tag{7}$$

The Cauchy stress tensor $\boldsymbol{\tau}$ for Williamson fluid is [38]:

$$\boldsymbol{\tau} = -\bar{\mathbf{p}}I + \bar{\mathbf{S}}, \tag{8}$$

$$\bar{\mathbf{S}} = \left[\mu_\infty + (\mu_0 + \mu_\infty)(1 - \Gamma\bar{\gamma}^*)^{-1}\right]\mathbf{A}_1, \tag{9}$$

in which $\bar{\mathbf{p}}$ is the pressure, $\bar{\mathbf{S}}$ the extra stress tensor, I the identity tensor, μ_∞ represents the infinite shear rate viscosity, μ_0 the zero shear rate viscosity, Γ the time constant. Also $\bar{\gamma}^*$ is defined as:

$$\bar{\gamma}^* = \sqrt{\frac{1}{2}\sum_i\sum_j \bar{\gamma}_{ij}^*\bar{\gamma}_{ji}^*} = \sqrt{\frac{1}{2}\Pi} \quad \text{and} \quad \Pi = tr(\mathbf{A}_1)^2, \mathbf{A}_1 = \text{grad } \mathbf{V} + (\text{grad } \mathbf{V})^T.$$

For $\mu_\infty = 0$ and $\Gamma\bar{\gamma}^* < 1$ the constitutive Eq. (9) takes the following form

$$\bar{\mathbf{S}} = \mu_0[1 - \Gamma\bar{\gamma}^*]^{-1}\mathbf{A}_1 = \mu_0[1 + \Gamma\bar{\gamma}^*]\mathbf{A}_1. \tag{10}$$

In above equations $\bar{\mathbf{V}} = (\bar{u}(\bar{r},\bar{z},\bar{t}), 0, \bar{w}(\bar{r},\bar{z},\bar{t}))$ represents the velocity, $\frac{d}{dt} = \frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial \bar{r}} + \bar{w}\frac{\partial}{\partial \bar{z}}$ the material time derivative, μ_0 the fluid viscosity, ρ_f the density of fluid, $(\rho c)_p$ the effective heat capacity of nanoparticle material, $(\rho c)_f$ the heat capacity of the fluid, k the thermal conductivity, $\bar{T}, \bar{C}$ the fluid temperature and concentration, $\bar{T}_m$ the mean temperature of fluid, c_p the specific heat at constant pressure, $D_{\bar{B}}$ the Brownian diffusion coefficient, $D_{\bar{T}}$ the thermophoretic diffusion coefficient, $\alpha_f = \frac{k}{(\rho c)_f}$ the ratio between the thermal conductivity and the heat capacity of the fluid and $\tau = \frac{(\rho c)_p}{(\rho c)_f}$ the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid, $\sigma$the electrical conductivity of fluid, $\bar{\mathbf{R}}$ the Darcy resistance in porous medium and $\bar{R}_r, \bar{R}_z$ the components of $\bar{\mathbf{R}}$ in radial and z-directions. Relation between pressure drop and velocity has been expressed by Darcy law. For Williamson fluid, the