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Finite amplitude cellular convection under the influence of a vertical magnetic field

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ABSTRACT

At the onset of stationary convection, the effect of a vertical magnetic field on the heat transfer of Rayleigh-Benard convection for an electrically conducting fluid is studied. The nonlinear governing equations describing the motion, temperature and magnetic fields are expanded as the sequence of non-homogeneous linear equations, which depend on the solutions of the linear stability problem. Infinite number of steady state with finite amplitude solutions are obtained for the stress-free boundary conditions. The perturbation method proposed by Kuo (1961) is used for the first time to highlight the heat transfer features of magnetoconvection. An explicit expression at the onset of convection in terms of parameters of the system is obtained. The dependence of heat transfer rate on Rayleigh number (R), Chandrasekhar number, thermal and magnetic Prandtl numbers is extensively examined until sixth order using an expansion of R as proposed by Kuo (1961). The results show that the magnetic field dampens the heat flow for stationary convection, i.e., the onset of convection shifts to higher values of R as the vertical magnetic field increases. Under the uniform magnetic field, heat flow gets enhanced as the thermal Prandtl number increases, whereas heat flow diminishes for the increase in magnetic Prandtl number. The results of flow field and heat transfer characteristics are depicted in the form of streamlines and isotherms, respectively. The presence of magnetic field changes the flow structure of streamlines from unicellular to multicellular patterns. This is due to the magnetic susceptibility of colder fluid flow towards the magnetic field. The flow field is analyzed with respect to the topological invariant relation. To trace the path of convective heat transport, the concept of Heatfunction has been employed. This methodology explains the comprehensive interpretation of energy distribution in terms of heatlines.

1. Introduction

An electrically conducting fluid confined between two horizontal plates and heated from below in the presence of a vertical magnetic field is the simplest geometry to study the magnetoconvection. The interaction between the thermal convection and the magnetic field, known as magnetoconvection, produces the Lorentz force, which resist the horizontal motion of fluid. Such type of motion occurs at the onset of natural convection. This convection with isothermal boundaries and no internal heat sources is governed by the non-dimensional parameter, namely, the Rayleigh number, R. There are two aspects of interaction between the magnetic field and the convection: on the one hand, the motion sweeps the magnetic flux aside and concentrates it in the isolated tubes or sheets; on the other, the Lorentz force affects, and may suppress the pattern of convection, thus, the fluid motion is more stable. Also, due to the presence of this force the corresponding R for the onset of convection increases.

The early research interest of magnetoconvection is mainly motivated by the geophysical and astrophysical applications and in particular the study of the existence of sunspots in an imposed magnetic field. It is necessary to understand the effects of the Lorentz force in convective motions of many astrophysical and geophysical problems. At the surface of the Sun, magnetic fields have high intermittent structures. Most of the flux is confined to tubes in which these fields are intense; in larger features, such as sunspots, heat transport is partially inhibited by the magnetic field.

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Nomenclature

A	amplitude
a	wavenumber
a_o	critical wavenumber
d	depth of the convection zone
$\mathbf{g}$	gravitational field vector
$\mathbf{H}$	external magnetic field vector
H_0	external magnetic field along Z-axis
H	Heatfunction
$\mathrm{H}_X, \mathrm{H}_Y, \mathrm{H}_Z$	components of magnetic field
$\mathbf{J}$	current vector
$\mathcal{L}$	linear operator
$\mathcal{N}$	nonlinear operator
N	Nusselt number
$N^{(2)}, N^{(4)}, N^{(6)}$	second, fourth, sixth order Nusselt numbers, respectively
N_L	local Nusselt number
P	effective pressure
p	growth rate
Pr_1	thermal Prandtl number
Pr_2	magnetic Prandtl number
Q	Chandrasekar number
R	Rayleigh number
R_s	stationary Rayleigh number
R_{oc}	critical oscillatory Rayleigh number
R_o	critical stationary Rayleigh number
T	temperature
T_s	static temperature
T_o	reference temperature
ΔT	temperature difference between upper and lower layers
t	time
$\mathbf{V}$	velocity vector
u, v, w	velocity components along X, Y, Z directions, respectively

Greek symbols

α	coefficient of thermal expansion
β	adverse temperature gradient
κ	coefficient of thermal diffusivity
ν	viscosity
ρ_o	reference density
μ_m	magnetic permeability
η	magnetic diffusivity

Superscript

$'$	variables with dimension

Generally cosmical magnetic fields are associated with turbulent motions, which may be due to convection. Similar theoretical studies of magnetoconvection within the framework of Boussinesq approximation have got inspired primarily by the existence of sunspots such as the formulation of linear problem at the onset of convection [2–6]. These studies motivated several researchers to investigate the suppression of convection by strong magnetic field [7–12]. The potential use of a magnetic field to control fluid flow and heat transfer in conductive fluids has long been recognized in many applications such as crystal growth, metal casting, liquid metal cooling blankets for fusion reactors, electric propulsion for space exploration, electronic packages, microelectronic devices and others. The liquid metals are characterize by their much larger magnetic diffusivity (η) in comparison with their thermal diffusivity (κ), i.e., in the limit of $\eta \gg \kappa$. These metals are used to study the low Prandtl number fluids ($Pr_1 \simeq 10^{-2}$). In these fluids, the two dimensional steady roll structures at the onset of convective flow easily become time dependent just above the critical stationary Rayleigh number, R_o. Apart from the low Prandtl number fluids recently several authors have investigated the effect of magnetic field on nanofluids [13–19].

Usually magnetic field effect is governed by the Chandrasekar number, Q. Using the linear stability analysis, Chandrasekar [2,3] showed that the R_o, increases with the increase in Q. He also discussed another important effect of vertical magnetic field, namely, dampening of the rate of heat transfer with the increase of magnetic field. These results are experimentally confirmed by Nakagawa [4,5].

Busse and Clever [10] discussed theoretically the stability of convective roll in the presence of a vertical magnetic field with rigid-rigid boundary conditions, for low Pr_1. It is observed that the presence of magnetic field tends to increase the efficiency of convective heat transport and thereby compensating in part for the delay at the onset i.e., of convection caused by the stabilizing effect of the Lorentz force. These results are experimentally obtained by Ulrich Burr and Ulrich Muller [20]. Also it is observed that the fluctuations in the temperature field get damped significantly by the presence of magnetic field. Cioni et al. [21] confirmed that their experimental threshold of convection was in agreement with linear stability theory [2,3] until $Q \cong 4 \times 10^6$. This Q value is higher than that of the previous experiments carried out by Nakagawa [4,5]. The main focus of the experiments was the characterization of turbulent magnetohydrodynamics (MHD) regimes occurring beyond the threshold. They have characterized two convective regimes influenced by the MHD effect, namely, in one regime the heat transfer rate or Nusselt number, N, is proportional to R, whereas in the second regime for the higher $R, N \cong R^{0.43}$. The measurement of N as a function of R and Pr_1 in cylindrical rolls for heat transport in turbulent RBC experimentally studied by Ahlers et al. [22]. It is shown that when R is constant, N varies with Pr_1 only by about two percent in the range of $4 < Pr_1 < 34$. This result contradicts with previous studies of Grossmann and Lohse [23] who have shown that this variation is about 20%. Most of the experimental studies on liquid metal convection have focused on the measurement of heat transport of the system and the temperature fluctuations at different points in the liquid layer related to the turbulence [24–26].

It can be noted that the above authors have studied magnetoconvection with rigid-rigid boundary conditions. Many researchers such as Biki and Karakisawa [27], Knobloch et al. [28], Knobloch and Proctor [29], Proctor and Weiss [30], Knobloch and Weiss [31] and Rucklidge [32] have studied a similar problem with stress free boundary conditions to explore the root of transition to chaos with respect to N. They derived a fifth-order ordinary differential equations model from a Galerkin truncation of the relevant partial differential equations and studied its properties both analytically and numerically for $Pr_2 > Pr_1$ and $R_{oc} < R_o$. Knobloch, Proctor, Rucklidge, and Weiss (hereafter referred to as KPRW) have criticized the work done by Biki and Karakisawa (hereafter referred to as BK) [27]. The response of BK on criticisms of KPRW is published [33]. On the other hand BK have also criticized the work of KPRW. The response for these comments by KPRW is also published [34].

Theoretically Malkus et al. [35] expanded the nonlinear equations describing the fields of motion and temperature in a