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A note on fractional order in thermo-elasticity of shape memory alloys' dampers

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ABSTRACT

In this paper vibration damping capacity of shape memory alloys (SMA) is studied, whose constitutive behavior is based on two dimensional Oberaigner, Fischer and Tanaka model (Oberaigner et al., 2002). The active and passive vibration damping paradigms of SMA with respect to thermal changes are touched here. In this work time-fractional order is given here. The above model solution is presented here for the most general case of a set of initial and boundary conditions which deal with generalized mechanical loadings and thermal regimes. For the particular cases, four examples for different thermal and mechanical loading-stresses, endorsed on the basis of fractional order with Homotopy Perturbation Method (HPM), are given. The solution given in this paper tells us how vibration response is studied with different thermal and different types of static and dynamic mechanical loading scenarios.

1. Introduction

Shape memory alloys (SMA) are amongst those functionally graded materials that possess 'shape recovery' properties based on phase shift mechanism [1–6]. In these alloys, thermionic states and temperature regimes coupled with induced strain are important pillars in defining the thermo mechanical behavior of SMA. SMA possesses two different thermodynamic-dependent behaviors beacuse of two phases; one lower temperature phase, martensite, generally described by M and second higher temperature phase, austenite described by A. Four transition-temperatures; the austenitic finish, A_f the austenitic start, A_s, the martensitic start, M_s and the martensitic finish, M_f are effective in thermo-elstic behavior of SMA. The relatively higher temperature austenite phase transforms to martensitic phase from higher temperature level A_f to lower temperature M_f [1,2]. The applied stress is also responsible to creation of stress induced martensitic micro-structure in SMA. However, the presence of both the phases is due to existence of inherently twinned-martensitic micro-structure [3]. These alloys have a tendency to show two different thermo-mechanical behaviors and outputs that largely depend upon thermodynamic states. Out of above discussed four phases, first one exists at the temperature near or above its A_f level, known as Austinitic phase. At Austinitic phase SMA can regain its original geometry soon after removal of applied stress without provision of external thermal energy and thus SMA behaves as perfectly pseudo-elastic material and used in passive devices. The pseudo-elastic and passive thermo-mechanical behavior of SMA works at a range of temperatures rather than any fixed temperature level [3–5]. The second behavior os SMA is known as shape memory effect (SME). At the temperature level lower than A_s, SME is possible only by providing the external thermal energy and enables SMA to recover its original geometry [6–9].

Shape Memory Alloys, can negotiate and support a wide range of applications and designs [10–15] because of its multi-functional characteristics [3], demonstrating both the sensing and actuation capabilities. It is being used in large scale structures. The unique thermo-elastic behavior of SMA makes it possible for being used as a plausible candidate in preservation and safety of the ancient buildings [12] and modern seismic resistant structures [6,8,9] because of its propensity to affirm the superelastic behavior against the bending [3,12], tension [6] and torsional [5] loadings as well as its greater capacity to withstand dynamic loadings and thus clinching an important character in the modern innovative technologies in smart devices and for the earthquake engineering.

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It is also reported that SMA has higher actuation energy density and higher actuation frequency[3] so it is also being used in micro-devices actuators with temperature based control system and in intelligent systems. Whereas, if there is an arrangement to have variable thermal settings, where temperature can be varied and controlled, SMA damper can be utilized as active device as stated in [16–19]. In the context of presence of different phases at different thermodynamic states, the relation between temperature and mechanical properties has been investigated. Furthermore, Dolce and Cardone [5,6] and Tamai [8] et al. studied the effects of temperature changes on SMA as they report that the critical stress that initiates the process of phase transformation in SMA, changes noticeably with temperature variations. Dolce and Cardone have alos observed the phenomenon of partial pseudo elasticity at temperature lower than A_f. Strandel et al. [20] studied the influence of temperature on thermo-mechanical respon se of SMA. Bhattacharyya [15] et al., investigated thermal effects on SMA actuators. Grant et al.[17,18] studied the control of active behavior of SMA. Lagoudas et al. [19] used electric current density as thermal state.

Fractional calculus is an effective method to alter and aspire modifications in many existing models of physical processes especially in physics and fluid mechanics and heat transfer and viscoelasticity [21–24]. In this work, time-fractional order for damping behavior of SMA is presented on the basis of the model [1], four sets of initial and boundary conditions are selected. Here, an effort is made to present a model that is intended to help in the design the SMA based systems for smart structural and actuator applications. The main purpose of this paper is to investigate the mechanical response of SMA membrane on the basis of temperature induced solid-solid phase transformations; subjected to different types of loadings and pinned to both ends. The system is generalized through the wave equation, the form of hyperbolic differential equation. The above mentioned studies [1–6] show the impact of temperature variations on SMA damping response, but their studies were related to some fixed quantity of temperature, model would be capable for using it for fixed quantities of temperature as well variable thermal states. Furthermore, we proposed some changes in the model, where we are going to modify the temperature field, and are taking temperature as function of time. This modification based on thermo-viscoelastic behavior of SMA would be affecting the utility of the model with respect to its usage as both active and passive systems and application of model is broadened as in the original model given in [1] was confined to passive systems only, so our modification would change its behavior into the multidisciplinary pattern, now our modification would make it capable for using in both active and passive systems.

The solution of time-fractional model provides a viable method in illustrating that how vibration is minimized when temperature field is affecting the phase transition with different thermal and different types of static and dynamic mechanical loading scenarios. The paper is organized in four sections; in first section we introduced the basic framework of Homotopy Perturbation Method (HPM). In the Section 2, we would find the two dimensional analytical solution of selected model with modification. In the 3rd section, four cases with different loadings conditions are discussed. In the last section, conclusions of the work are given.

2. Problem formulation and solution

Consider the two dimensional membrane of SMA and it is represented in mathematical form of the Oberaigner et al., model, given in [1].

The constitutive law discussed here relates stresses and transformation of phases with uniformly distributed heat transfer. The mathematical model accommodates a highly coupled thermodynamic and mechanical loading correlation. Here, the hyperbolic differential equations show the phenomenon of thermo-viscoelastic behavior of SMA by incorporating the equations of heat transfer also.

$$\frac{\partial u^{\delta}}{\partial t^{\delta}} - \alpha^2\left(\frac{\partial u^2}{\partial x^2} + \frac{\partial u^2}{\partial y^2}\right) + \beta^2(Q(x,y,t)) = 0 \tag{1}$$

with boundary conditions

$$\begin{aligned} &u(x,0,t) = 0, \quad u(x,H,t) = 0,\\ &u(0,y,t) = 0, \quad u(L,y,t) = 0,\\ &u(x,y,1) = 0, \quad m(x,y) = 0 \end{aligned} \tag{2}$$

and initial conditions as

$$u(x,y,0) = k(x,y), \quad \frac{\partial u(x,y,0)}{\partial t} = 0, \quad 0 \leqslant x \leqslant L, \quad 0 \leqslant y \leqslant H \tag{3}$$

Homotopy Perturbation Method is selected for obtaining solution of above model.

2.1. Homotopy Perturbation Method (HPM)

The principals of the HPM and its applicability for various kinds of differential equations are given in [25–30]. For convenience of the reader, we will present a review of the HPM [31–36] then we will present the algorithm of the new modification of the HPM [37–41]. To achieve our goal, we consider the nonlinear differential equation

$$L(u) + N(u) = f(r) \tag{4}$$

with the boundary conditions

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad r \in \Gamma \tag{5}$$

where L is a linear operator, while N is nonlinear operator, B is a boundary operator, Γ is the boundary of the domain Ω and $f(r)$ is a known analytic function. The He's homotopy perturbation technique [25–30] defines the homotopy $v(r,p) : \Omega * [0,1] \longrightarrow \Re$ which satisfies

$$H(v,p) = (1-p)[L(v) - L(u_\circ)] + p[L(v) + N(v) - f(r)] = 0 \tag{6}$$

or

$$H(v,p) = (1-p)[L(v) - L(u_\circ)] + p[L(u_\circ)] + p[N(v) - f(r)] = 0 \tag{7}$$

where $r \in \Omega$ and $p \in [0,1]$ is an impeding parameter, u_o is an initial approximation which satisfies the boundary conditions.

Obviously from Eqs. (6) and (7) we have

$$H(v,0) = [L(v) - L(u_\circ)] = 0 \tag{8}$$

$$H(v,1) = [L(v) + N(v) - f(r)] = 0 \tag{9}$$

The changing process of p from zero to unity is just that of $v(r,p)$ from $u_\circ$ to $u(r)$. In topology, this called deformation, $[L(v) - L(u_\circ)]$ and $[L(v) + N(v) - f(r)]$ are homotopic. The basic assumption is that the solution of Eqs. (6) and (7) can be expressed as a power series in p:

$$v = v_o + pv_1 + p^2 v_2 + \cdots \tag{10}$$

The approximate solution of Eq. (7), therefore, can be readily obtained:

$$u = \lim v_{p\to 1} = v_\circ + v_1 + v_2 + \cdots \tag{11}$$

The convergence of the series Eq. (11) has been proved in[25–30].